POSITIVE AND NEGATIVE PROPERTIES.
A LOGICAL INTERPRETATION

Abstract
In the paper we construct a simple sentential logic \( L_{BK} \) based on the ontology presented in [3]. \( L_{BK} \) has internal and external strata, which yield a double characterization of the connectives. This leads to the correspondence of \( L_{BK} \) to Bochvar’s and Kleene’s logics. The connectives of the internal level correspond to the connectives of Bochvar’s internal logic and of Kleene’s weak logic, while the connectives of the external level correspond to those of Bochvar’s external logic and of Kleene’s strong logic. The ontological interpretation shows that the former represent the so-called connectives "de re", the latter the connectives "de dicto".

1. Inspirations
We present a particular problem concerning properties of individuals. Given some representation of statements about individuals by the language of many-valued logic we show simple relations that connect objects and their properties. The problem is a piece of more general topic of formal representation of three categories of objects: universals (that are treated as incomplete objects), individuals (complete objects) and concepts (that are treated by philosophers as incomplete objects; we do not investigate them here).

For our analysis we distinguished two types of objects: complete and incomplete. Complete objects can be identified with individuals (like
Socrates, Tarski) and incomplete objects with universals (man, horse). The division and terminology are attributed to Meinong [7]. According to him we can also distinguish properties such as redness and compliment properties such as non-redness. So, if redness belongs to an object, then redness is its positive property. Hence, non-redness is its negative one.

In accordance with classic ontological approach (Aristotle, medieval philosophers, Wolff) the individuals we talk about are described by these properties. Philosophers point out that one can distinguish three kinds of properties in objects: essential, attributive and accidental properties. Rational is an essential property of the man Socrates, being able to smile is his attribute and being dark-haired is his accidental property. Taking into account the division of properties into positive and negative we propose to say that rational is a positive and essential property of an individual. Non-rational would be an essential and negative property of that individual. Next, if all animals are non-rational then non-rational is positive and (perhaps) essential property of the individual horse. Similarly, in the case of Socrates being a lawyer is his negative and accidental property.

For Meinong and Łukasiewicz [4] any property P is a positive or negative property of an object a. In this paper we propose to limit this thesis by showing it appears evident that the properties odd or even can not be referred to a man. It means that the sentences:

1. Socrates is even,
2. Socrates is not even (is odd)

are neither true nor false. We will call this class of properties non-related (or non-connected) properties of an object a. Then the remaining properties, i.e. the properties which apply to a, build a class of objects that can be naturally divided into three subclasses: essential, attributive and accidental properties. We assume that the Meinong’s thesis is true for the second class, the class of related properties. So, for example, if the property of being just belongs to the class of related properties of Socrates, then if the sentence

3. Socrates is just

is true, then

4. Socrates is unjust (non-just)

is a false sentence.

Individual objects are correlated with universals i.e. with incomplete objects. In [3] we presented the idea, which states that universals are composed as a hierarchical structure, which we called Porphyrian Tree.
Structure (PTS). Any object of the structure is not described by a property of individuals. For example it is evident that universal man does not possess the property of being rational. However the universal can be instantiated by a number of individuals ([11]). This means that some characteristic of a given individual “stick” in the universal. Ingarden, for example, states that an idea has a double-stratum construction and one of the strata establishes the content of individuals [2]. Hence, we define a universal, any element of PTS, as a pair $<f, g>$ of functions, where $f$ and $g$ are functions of $T'$ – a subset of set of $T$ properties – into $\{0, 1/2, 1\}$. We assume that the $f$ represents essential properties and $g$ represents attributes. In the case of analysis of the universal man, according to intended interpretation: $f(\text{rational}) = 1$, so, \text{rational} is essential positive property belonging to the content of universal man, $g(\text{inability to smile}) = 0$, so \text{inability to smile} is negative attribute of the content and if $g(\text{being white}) = 1/2$ then \text{being white} is accidental property belonging to the content of universal [3].

The representation of universals outlined above is a starting point for constructing a semantic structure. The propositional terms and formulas that express properties of individuals we can interpret in the structure given below (cf. Point 3). In this paper we only analyse simple formulas, which are the counterparts of simple sentences of natural languages e.g. Socrates is just. At the same time we focus our interests on positive and negative properties of individuals. We take into account a three-valued logic (Bochvar’s and Kleene’s logic) in order to show that connectives on the ontic side (called \text{de re}) operate the same as those of three-valued logic (cf. Point 2, 3).

We distinguish two logics: $L_{INT(BK)}$ and $L_{BK}$ (corresponding to two levels of language) the internal of elementary propositional terms containing the \text{de re} negation and other \text{de re} connectives and the external language of formulas with the \text{de dicto} negation and other connectives. Their properties and the fact that they are coherent with Bochvar’s and Kleene’s logics indicate that three-valued logic has also its foundations in ontologically interpreted objects (see Point 4.1).

In this sequel we examine the above-mentioned matter by considering a simple sentential language and constructing a semantically defined logic.
2. Language \( L \)

\( L \) is a two-level language. The first level comprises propositional terms: atomic and elementary. The elementary propositional terms constitute the base for the language i.e. the second level. By inserting an elementary propositional term into parentheses we obtain an elementary formula of \( L \) and by using connectives we construct sentences of \( L \). Propositional terms and sentences are differentiated as in Theory of Proposition (cf. [10]).

A. Vocabulary
We define \( \text{Const} = \{a_1, a_2, \ldots\} \) as a set of individual constants and \( \text{Prop} = \{t_1, t_2, \ldots\} \) as a set of the names for properties. We consider logical constants: \( \neg, \land \) as de re and \( \neg, \land \) as de dicto constants. The connectives \( \lor, \rightarrow, \leftrightarrow \) can be introduced by abbreviations in the standard way.

B. Propositional terms and formulas.

The set of atomic propositional terms \( \text{L}_{ap} \) and the set of elementary propositional terms \( \text{L}_{ep} \) are defined by induction:

2.1. If \( t \in \text{Prop} \) and \( a \in \text{Const} \), then \( t(a) \in \text{L}_{ap} \) and \((\neg_r t)(a) \in \text{L}_{ap}\).
2.2. \( \text{L}_{ap} \subseteq \text{L}_{ep} \)
2.3. If \( \alpha, \beta \in \text{L}_{ep} \), then \( \neg_r \alpha \in \text{L}_{ep} \) and \( \alpha \land \beta \in \text{L}_{ep} \).

Remark 1. In the case \( \alpha \in \text{L}_{ap} \), and \( \alpha \) has a form \( t(a) \) or \( (\neg_r t)(a) \), we will use the sign \( \sim \) and write the sentences \( \neg_r t(a) \) and \( \neg_r (\neg_r t)(a) \) as \( t \sim (a) \) and \( (\sim t) \sim (a) \), respectively. To explain this let us take into account sentences of natural language and its formal counterparts. Let \( a \) be the counterpart of Socrates and \( t \) of just.

1'. Socrates is just. \( s(a) \)
2'. Socrates is not just. \( s \sim (a) \)
(we propose this notation for the sentence which is opposite to (1'))
3'. Socrates is unjust. \( (\sim s)(a) \)
4'. Socrates is not unjust. \( (\sim s) \sim (a) \) (consequently to (2')).

Next, the set \( L \) of formulas is defined by the conditions:

2.4. If \( \alpha \in \text{L}_{ep} \), then \( (\alpha) \in \text{L}_c \)
2.5. \( \text{L}_c \subseteq \text{L} \)
2.6. If \( \alpha, \beta \in \text{L} \), then \( \neg \alpha \in \text{L} \) and \( \alpha \land \beta \in \text{L} \).
Remark 2. Parentheses ( ) given in 2.4 are the counterpart of Bochvar’s sign $A^*$, i.e. of external assertion (cf. [8]).

3. Model structure $M$ and Logic $L_{BK}$.

A. The model structure $M$ is given as triple:

$$M = <PTS, U, | >,$$

where:

- $PTS$ is a PTS structure for a set of properties $T$,
- $U$ is any non-empty set of complete objects given in the following way:
  
  Let $G = \{G_1, \ldots, G_n\}$ be a set of natural species for the PTS. For any $G_i = <e_i, a_i>, 1 \leq i \leq n$, let us consider a set $OB_i$ defined as:

  $$OB_i = \{o : D(o) = D(a_i) = T', T' \subseteq T, \text{ and } o(T') \subseteq \{0, 1\}\}.$$

  Comment: $D(f)$ is the domain of function $f$; the closing condition shows that positive (negative) properties of incomplete objects (precisely: properties of the content of incomplete objects) are positive (negative) properties of complete objects. Then, let us put

  $$U = \bigcup_{i=1}^{n} OB_i$$

Example. Let us consider a simple PTS structure:

$$<e_0, a_0>$$

$$<e_1, a_1>$$

$$<e_2, a_2>$$

$$<e_3, a_3>$$

$$<e_4, a_4>$$

$$<e_5, a_5>$$

$$<e_6, a_6>$$

$$<e_7, a_7>$$
For $i = 1, 3, 4, 6$ and $7$ the incomplete object $< e_i, a_i >$ is a natural species.
Let us assume that $a_7$ is a function on $T^*$, where $T^* = \{ t_2, t_4, \ldots, t_{2n} \}$
and $a_7(t_2) = a_7(t_3) = a_7(t_4) = a_7(t_6) = 1$ and, for $n \geq 5$, $a_7(t_{2n}) = 1/2$.
Then, a complete object $o$ such that for $n < 5 o(t_{2n}) = a_7(t_{2n})$ and for
$n \geq 5 o(t_{2n}) \in \{ 0, 1 \}$ is an example of object of $OB_7$ which is generated by
the incomplete object $< e_7, a_7 >$.

- $| |$ is a function on $\text{Const} \cup \text{Prop} \cup L$ satisfying the conditions:

1. For any $a \in \text{Const} : |a| \in U$.
2. For any $t \in \text{Prop} : |t| \in T$.
3. For elementary propositional terms $L_{ep}$ function $| | : L_{ep} \to \{ 0, 1, 2, 1 \}$ is defined by the conditions ($\nu$ stands for $| |$):

Negation (de re)

$(T \sim 1) \nu(s(a)) = 1$ iff $|s| \in |a|^{-1}(\{0, 1\}) \& |s| \in |a|^{-1}(\{1\})$.
$(T \sim 2) \nu(s \sim (a)) = 1$ iff $|s| \in |a|^{-1}(\{0, 1\}) \& |s| \not\in |a|^{-1}(\{1\})$.
$(T \sim 3) \nu((\sim s)(a)) = 1$ iff $|s| \in |a|^{-1}(\{1, 0\}) \& |s| \in |a|^{-1}(\{0\})$.
$(T \sim 4) \nu((\sim s) \sim (a)) = 1$ iff $|s| \in |a|^{-1}(\{0, 1\}) \& |s| \not\in |a|^{-1}(\{0\})$.

$(F \sim 1) \nu(s(a)) = 0$ iff $|s| \in |a|^{-1}(\{0, 1\}) \& |s| \not\in |a|^{-1}(\{0\})$.
$(F \sim 2) \nu(s \sim (a)) = 0$ iff $|s| \in |a|^{-1}(\{0, 1\}) \& |s| \not\in |a|^{-1}(\{0\})$.
$(F \sim 3) \nu((\sim s)(a)) = 0$ iff $|s| \in |a|^{-1}(\{0, 1\}) \& |s| \not\in |a|^{-1}(\{1\})$.
$(F \sim 4) \nu((\sim s) \sim (a)) = 0$ iff $|s| \in |a|^{-1}(\{0, 1\}) \& |s| \not\in |a|^{-1}(\{1\})$.

$(1/2) \nu(a) = 1/2$ in other cases, i.e. if $|s| \not\in |a|^{-1}(\{0, 1\})$ and $a$ is of
one of the four forms.

Remark 3. In $(T \sim 1)$ and $T \sim 3$ the condition $|s| \in |a|^{-1}(\{0, 1\})$ can be
omitted. The condition is essential for $(T \sim 2)$ and $(T \sim 4)$. It indicates
that the sentences of the forms: $s(a)$ and $(\sim s) \sim (a)$, $((\sim s)(a))$ and $s \sim (a)$,
respectively) are equivalent. An analogous remark holds the same for the
conditions $(F \sim 1) - (F \sim 4)$ i.e. $\nu(s \sim (a)) = 0$ iff $\nu((\sim s)(a)) = 0$ and
$\nu(s(a)) = 0$ iff $\nu((\sim s) \sim (a)) = 0$.

Remark 4. The condition $(1/2)$ is based on the idea that when $t$ does not
belong to the domain, the terms $t(a)$, $(\sim t)(a)$ are neither true nor false.
Conjunction (de re)

\[ (T \land_r) \nu(\alpha \land_r \beta) = 1 \text{ iff } \nu(\alpha) = 1 \text{ and } \nu(\beta) = 1 \]
\[ (1/2 \land_r) \nu(\alpha \land_r \beta) = 1/2 \text{ iff } \nu(\alpha) = 1/2 \text{ or } \nu(\beta) = 1/2 \]
\[ (F \land_r) \nu(\alpha \land_r \beta) = 0, \text{ in oth.} \]

3.4. The assignment \(|\ |\) is a function of \(L\) into \(\{0, 1\}\) and fulfils the conditions:

3.4.1. If \(\alpha \in L_{ep}\), then \(|(\alpha)| = 0 \text{ iff } \nu(\alpha) = 0 \text{ or } \nu(\alpha) = 1/2 \]
\[ |(\alpha)| = 1 \text{ iff } \nu(\alpha) = 1 \]

3.4.2. If \(\alpha \in L\), then \(|\neg \alpha| = 1 \text{ iff } |\alpha| = 0\) (negation de dicto)

3.4.3. If \(\alpha, \beta \in L\), then \(|\alpha \land \beta| = 1 \text{ iff } |\alpha| = 1 \text{ and } |\beta| = 1\) (conjunction de dicto).

B. The Logic \(L_{BK}\).

The logic is presented semantically in the standard way. Namely, a sentence \(\alpha \in L\) is true in a model structure \(M = < PTS, U, |\ |_M >\) for \(L\) if and only if for any \(|\ |_M : |\alpha|_M = 1\). A formula \(\alpha\) is logically true (in short: \(\models \alpha\)) if and only if it is true in all model structures \(M\) of \(L\). Finally, formula \(\alpha \in L\) is a semantic consequence of a set of formulas \(X \subset L\) (\(X \models \alpha\)) if and only if for any model \(M\) and any \(|\ |_M : \text{for any } \beta \in X : |\beta|_M = 1\), then \(|\alpha|_M = 1\). The logic will be denoted by \(L_{BK}\).

Remark 5. In our opinion the Conditions for de re and de dicto connectives result from the structure of objects. We are going to show that this leads to the three-valued Bochvar’s logic. To this aim let us recall the matrices that define the connectives of Bochvar’s internal and external logics (cf. [8]):

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External (classical) connectives

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4. Conclusions

4.1. The logic defined in 3.B is classical. Let us observe that the logic with de re connectives is three-valued (this being established by M and ν fulfilling the conditions given in Point 3). We will call this logic $L_{INT(BK)}$. It is evident that $L_{INT(BK)}$ is Bochvar’s internal logic. The connectives de re operate like the connectives on internal language of Bochvar’s logic and of weak Kleene’s logic (cf. [1], [6]). On the contrary, the connectives de dicto (on $L$) operate like the classical ones (cf. [8]). Does that mean that we should understand the de re negation in a three-valued way? In this paper we suggest so.

4.2. The four sentences (1′) – (4′) are related to some states of affairs or events on the ontic side. Perhaps, it would be more appropriate to use the sentences of the forms:

(1′) Socrates has the property of being just.
(2′) Socrates does not have the property of being just.
(3′) Socrates has the property of being unjust.
(4′) Socrates does not have the property of being unjust.

According to Remark 3 the sentences (1′) and (4′) and also (2′) and (3′) refer to the same events. Reinach ([9]) would grasp both events in the phrases: being just of Socrates (instead of (1′) and (4′)) and not-being just of Socrates ((2′) and (3′)). Let us also observe that the negation de re is used in (2′) and (3′). Why? We assume that this negation is a component of state of affairs and the object we talk about has a complementary property to a given one (cf. [4], p. 139). Let us notice, however, that $\overline{\sim s(a)}$ and $((\sim s)(a))$ are not equivalent in $L_{BK}$, so possessing negative properties by an object is not equivalent to the negation (de dicto) of the sentence in which one states that an object has a complementary positive property. Hence, negation de re (namely: negation on the ontic side) cannot be treated as classical.
4.3. Obviously, the law of the excluded middle holds in the given logic with external (classical) negation, i.e.

\[ \models \alpha \lor \neg \alpha, \]

but the sentence \( t(a) \lor \sim t(a) \) is not a law of \( L_{BK} \). In such a sentence we predicate \( t \) and its compliment \( \sim t \) about the object \( a \) (cf. [7] and [5], entry: Meinong). These formulas are the counterparts of the sentences given below.

(E1) It is true that Socrates is even or it is not true that Socrates is even.

(E2) Socrates is even or Socrates is not even.

In the intended interpretation (E1) is true and (E2) is false.

We propose to treat the two formulations of the law of the excluded middle as a representation of Meinong’s distinction between the narrower (\( \sim \)) and the wider (\( \neg \)) sense of negation and the narrower and the wider (classical) sense of the law of excluded middle (cf. [5], page 306).

On the other hand for conjunction, we obtain:

\[ \models \neg (\alpha \land \sim \alpha), \]
\[ \models \neg (t(a) \land_r \sim t(a)) \]
\[ \models \neg (t(a) \land \sim t(a)) \]
\[ \models (\sim (t(a) \land_r \sim t(a))). \]

5. Perspectives

Laws of classical logic are treated as equivalents of ontological laws (e.g. the ontological Law of Non-contradiction is a counterpart of the logical Law of Non-contradiction). Then it is possible to consider thesis of \( L_{INT(BK)} \) as counterparts of ontological ones. But it is a well-known fact that the set of theses of Bochvar’s internal logic and of Kleene’s weak logic is empty (see [8]) and then \( L_{INT(BK)} \) has no thesis. Hence, the question is arising if ontology with connectives "de re" has no laws? It appears that it is possible to reconstruct these logics and build some logic with a non-empty set of theses. To this end we extend language of \( L_{BK} \) to the language with monadic predicates and \( \iota \)-operator. Let us call \( L'_{INT(BK)} \), a logic upon
extended language that is associated with $L_{INT(BK)}$, and $L'_{BK}$, a logic associated with $L_{BK}$, respectively. Then, the elementary propositional terms of the form $R(ιR)$ which can be considered in $L_{INT(BK)}'$ and their counterparts $(R(ιR))$ in logic $L_{BK}$ are examples of logically true formulas. In the succeeding steps we will outline the proposed solution. We enrich the vocabulary by set PRED of 1-ary predicates $R_1, R_2, \ldots$. Next, we define a set of names for individuals: $NI = Const \cup \{ιr : r \in \text{PRED}\}$. Then, $R(a_1), R(ιR_1), t(ιR)$, etc. are some of the elementary propositional terms of $L_{ep}$. Let us consider a modified model for $L_{ep}$ in which we describe sense and denotation of predicates. In conformity with Meinong who treated incomplete objects as Platonic forms or ideas, we consider objects of PTS as senses of predicates (formally: $||R||$ is the sense of $R$), i.e.: for any $R \in \text{PRED}$:

$$||R|| \in \text{PTS}.$$  

Finally, the denotation of the name $ιR$ is an object of $|R|$. There $id_2(||R||)$ stands for the second element in a pair of a PTS; $ς_01$ is defined in [3] and expresses that each positive (negative) property described in $id_2(||R||)$ is positive (negative) property of $o$.

Let us put for any $r, r' \in \text{PRED}$:

$$\nu(r(ιr')) = 1 \text{ iff } (D(id_2(||r'||)) \subseteq D(id_2(||r||)) \text{ or } D(id_2||r||) \subseteq D(id_2||r'||) \text{ and } |r'| \in |r|,$$

$$\nu(r(ιr')) = 0 \text{ iff } (D(id_2(||r'||)) \subseteq D(id_2(||r||)) \text{ or } D(id_2||r||) \subseteq D(id_2||r'||) \text{ and } |r'| \not\in |r|,$$

$$\nu(r(ιr')) = 1/2 \text{ iff it is not true that } (D(id_2(||r'||)) \subseteq D(id_2(||r||)) \text{ or } D(id_2||r||) \subseteq D(id_2||r'||)).$$

Hence, for any model $M : \nu(R(ιR)) = 1$ and $|R(ιR)| = 1$. It means that the set of theses of $L_{INT(BK)}'$ is non-empty. We hope to give a more detailed analysis of the presented problem in a separate paper.

References


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