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THE COLLAPSE OF THE DESCRIPTIVE COMPLEXITY OF TRUTH DEFINITIONS.
COMPLETIONS OF HEYTING AND BOOLEAN ALGEBRAS

We are interested in applying constructive methods in (classical and intuitionistic) model theory. We describe in a canonical way and in the frame of constructive metamathematics a procedure for an embedding with a small descriptive complexity of arbitrary Boolean (or Heyting) algebra in a complete algebra, preserving all existing unions and meets. As a pre-history of the method we mention the work of Friedman (as it described in [1]) and the well-known lemma of Rasiowa-Sikorski about completing of Boolean algebras with preserving a given countable family of unions and meets.

As an application we get the following results about truth definitions with small descriptive complexity.

**Theorem 1.** Let $T$ be a r.e. classical axiomatic theory. A complete Boolean algebra $B$ (whose elements are sets of natural numbers) can be constructed and a $B$-valued model $M(B)$ can be defined (in a constructive way) such that:

(i) $n \in \|A\|_B$ is $\Pi^0_2$ predicate of $n$ and $A$;
(ii) $\|A\|_B = 1$ iff $T \vdash A$.

**Theorem 2.** Let $T$ be a r.e. intuitionistic axiomatic theory. A complete Heyting algebra $H$ can be constructed and an $H$-valued model $M(H)$ can be defined (in a constructive way) such that:

(i) $n \in \|A\|_H$ is $\Sigma^0_1$ predicate of $n$ and $A$;
(ii) $\|A\|_H = 1$ iff $T \vdash A$. 
Let us consider now second order Peano arithmetic $PA_2$.

**Theorem 3.** A complete Boolean algebra $B_1$ and corresponding $B_1$-valued model $M(B_1)$ for $PA_2$ can be defined such that:

(i) $n \in \|A\|_{B_1}$ is $\Pi^1_1$ predicate of $n$ and $A$;
(ii) $\|A\|_{B_1} = 1$ is a $\Pi^1_1$ predicate of $A$;
(iii) $M(B_1)$ is standard on natural numbers (i.e. $\|\forall x A(x)\| = \bigwedge_{n \in \omega} \|A(x)\|$ for all formulas $A$) and $M(B_1)$ is complete and correct for the elementary truth definition;
(iv) all sets in this model are definable in $M(B_1)$.

**References**


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