TANNENBAUM PHENOMENA FOR
BOUNDED EXISTENTIAL FUNCTION

I will present some new results concerning the Tennenbaum phenomena of the weak theory of first-order arithmetic $IE_1$ of bounded existential induction introduced by George Wilmers, and of finite subtheories of $IE_1$.

In particular I will present a simplification of the proof of Wilmers’ main theorem that all nonstandard models $M$ of $IE_1$ have reducts $M | +$ that are recursively saturated models of Presburger arithmetic (and hence these countable reducts are exactly the reducts $M' | +$ where $M'$ satisfies $PA$). This simplification will enable me to analyze in more detail the status of a problem posed by Wilmers’ (whether his theorem holds for some finite fragment of $IE_1$) and relate this question to problems in diophantine complexity.

I shall also investigate the Tennenbaum properties of e.c. models of certain finite fragments of $IE_1$, showing that these satisfy overspill for $\exists_1$ formulas, and have nonstandard initial segments satisfying $PA$. As corollary I will deduce that Hilbert’s 10th problem for $IE_1$ (to give a procedure to decide which diophantine equations have a solution in some model of $IE_1$) is actually insoluble.