ANSWERING ANOTHER ALLEGED DILEMMA DESTROYING DIALETHEISM

To leave matters in no doubt, we obligingly assert that the Russell class \( R \), i.e. \( \{ x : x \not\in x \} \), both belongs to itself and also does not belong to itself; in short, we assert \( R \in R \& \sim (R \in R) \). To be quite explicit, we assert the contradiction \( r \& \sim r \), where \( r \) abbreviates \( R \in R \). Thus, in convenient symbols, \( \vdash_{\delta} r \& \sim r \), where \( \delta \) is the group of dialethicians comprising (at least) Priest and Routley. Now Goldstein asserts not, or not just, that we should not do what we have naughtily done, but that we cannot; it “is not that people should not assert contradictions, but that they cannot, even though they may purport to do so” ([1], p. 11).

Goldstein offers a neat, but nonetheless fallacious, argument to support his assertion that we, along with distinguished dead dialethicians (Fichte, Hegel and Peirce are cited), cannot do what we purport to have done, asserted a contradiction. The argument, which yields “the lesson to be learnt from the problem about assertion” “for the dialetheist”, when more fully formalized using commonplace notation (of [2]), appears to go as follows:

1. \( \vdash_{\delta} (r \& \sim r) \), from some \( r \), by hypothesis.
2. \( \vdash_{\delta} r \& \vdash_{\delta} \sim r \), by the principle
   \( P(i) \vdash_{X} (A \& B) / \vdash_{X} A \& \vdash_{X} B \) (“whoever asserts a conjunction asserts each conjunction” in rule and generality form).
3. \( K_{X}(r \& \sim r \iff r \iff \sim r) \), for every assertor \( x \) with a “modicum of logical sophistication”, where \( K \) is an epistemic function of recognition.
4. \( \vdash_\delta (r \leftrightarrow \neg r) \), by 1, 3, the assumption that assertor(s) has a modicum of logical sophistication, and the principle
\[ P(iii) \quad K_X(A \leftrightarrow B), \vdash_X A \quad \vdash_X B \] (a much weakened version of Goldstein’s principle (AI)).

5. \( \vdash_\delta (r \rightarrow \neg r) \) & \( \vdash_\delta (\neg r \rightarrow r) \), for instance by a similar argument to that for 4, using the “recognized equivalence” \( (r \leftrightarrow \neg r) \leftrightarrow (r \rightarrow \neg r) \vee (\neg r \rightarrow r) \). (Goldstein omits this and the next obvious step.)

6. \( \vdash_\delta (r \rightarrow \neg r) \), simplifying 5 (either conjunct of 5 would do).

7. \( \sim \vdash_\delta r \) & \( \sim \vdash_\delta \neg r \), by the principle
\[ P(ii) \quad \vdash_X A \rightarrow \neg \neg_X A \& \neg_X B \] (whoever asserts a conditional does not assert the antecedent and does not assert the consequent).

8. \( \vdash_\delta r \) & \( \vdash_\delta \neg r \) & \( \vdash_\delta \sim r \) & \( \vdash_\delta \sim r \), by 2 and 7.

But, asserting both \( r \) and \( \sim r \) and not asserting either (“acknowledging the truth of neither”) is “too rich even for” dialetheicians.

There are various impeachable points in this argument, or Goldstein’s informal version of it, which we shall simply note and let pass. Principle \( P(i) \) slipped by, like \( P(ii) \), as “generally thought to be uncontroversial”, is disputed, not only in holist enclaves and the like, but by those who (erroneously) contract assertion to utterance. However for the minimal two member set \( \delta = \delta(min) \) it is not disputed; that is, we accept \( P(i) \). But though we grant \( P(iii) \) for a good (non-material) equivalence, i.e. \( x \) can be instantiated to \( \delta(min) \), we do not concede 3. For \( A \& B \rightarrow A \leftrightarrow B \) fails for good implication or conditional. (Also we would certainly want to reject Goldstein’s (AI). The same (type) assertion is made by the case of recognizedly equivalent sentences,

and his specious argument for it; but it can be dispensed with in favour of the weaker, acceptable \( P(iii) \), except for a residual role in applying \( P(ii) \).)

These issues are not particularly germane to the argument presented because we are prepared to assert, on independent grounds, both \( r \leftrightarrow \sim r \) and \( r \rightarrow \sim r \) (though not as the same assertions); that is, both 4 and 6 hold for \( \delta = \delta(min) \). Nor will we delay further what we see to be the main present issue by plumping for 8. Some dialetheicians may want to deny that assertion has a consistent logic; but we do not need to, or to pursue such riches here.
Richard Sylvan and Graham Priest

The crucial issue, the only thing left really to contest, is principle $P(ii)$. It is false. What is correct (to cast the matter in stronger implication form) is not $\vdash_X (A \rightarrow B) \rightarrow \neg \vdash_X A$, to take the antecedent case, but rather $\vdash_X (A \rightarrow B) \not\vdash_X A$; but the first does not follow from the second (pace Boethius). What seems to displace $P(ii)$ is, to forge new notation,

$$P(ii)' \vdash_X (A \rightarrow B) \not\vdash_X A; \vdash_X (A \rightarrow B) \not\vdash_B,$$

i.e. there is no direct inferential way from assertion of a conditional to assertion of its antecedent or its consequent. This begins to capture what is correct about the ambiguous “Frege point”, “to assert a conditional is to assert neither antecedent nor consequent” (p. 10). Of course someone also asserts a conditional may also assert one or more components, as we do with 6; but we are not in generally logically committed by the conditional itself to that. With $P(ii)'$ in place of $P(ii)$, steps 7 and 8 no longer follow; the argument collapses.

Now Goldstein has responded (by letter) that his argument should be differently formalized, that it is perverse to represent through the obviously false $P(ii)$ the principle that he intended to embrace: “whoever asserts a conditional does not thereby assert the antecedent or consequent” ($PG$). Then, Goldstein claims, “the “problem for the dialetheist” remains”. Does it? The general form of the destructive argument is this: There are hypotheticals (conditionals or biconditionals) which entail or coentail assertorics with common components. Let $Hy, As$ and $Cc$ represent, respectively, such statements; Goldstein’s working example concerns $A \leftrightarrow \neg A$ and $A \& \neg A$, with components $A$ and $\neg A$, but there are other significant examples, such as $\neg A \rightarrow A$ and $A$. Now, according to $PG$, those (e.g. $\gamma$) who assert cases of $Hy$ are not thereby asserting the component $Cc$. Nor according to a dubious commitment variant $PG^C$ of $PG$ (applied by Goldstein) are they thereby committed to asserting $Cc$. But those who assert $Hy$ are, if not asserting, at least committed to asserting what it entails $As$, and so committed to asserting its component $Cc$ (by virtue of inclusion or entailment).

In sum

$$\vdash_{\gamma} Hy \not\vdash_{\gamma} Cc \text{ and } \vdash_{\gamma} Hy \vdash_{\gamma} Cc,$$

where symbol $\vdash$ represents either implication, yielding, or commitment to, depending upon how the argument is duly filled out. But this is impossible, and is again “too rich” a contradiction.
The trouble with this little argument is that it achieves too much, putting mainstream theorists in a decidedly more embarrassing position than dialetheicians. For let \( A \) be any assertoric necessary truth, for instance \( \Box B \) (to avoid some quibbles) with component \( B \). Then according to mainstream theory, \( A \) coentails the hypothetical \( \sim B \rightarrow B \). Should mainstream theorists assert any such necessary truth, for instance in the course of using a bit of mathematics, then they are stuck and also not stuck with one and the same component. Mainstream mathematics would trivialise by virtue of its surroundings, its assertoric epitheory.\(^1\) By contrast however with the logico-semantical paradoxes, such arguments to inconsistency are patently bad. For principles \( PG \) and \( PG^C \) are incorrect without significant modal or quantificational qualification. In particular, in certain relevant cases, though not in general, one who asserts \( H_y \) is committed to asserting \( Cc \). For instance, a mainstream theorist who asserts \( \sim B \rightarrow B \) is committed to asserting \( B \). But once requisite disambiguating qualifications are made to the principles none of various arguments, whether through assertion or differently through commitment or obligation to assert, any longer succeed. The apparently robuster cases Goldstein would rely upon can be all decently accounted exceptions to his tattered principles.

Nor can a Goldstein argument be reinstated by principles dialetheicians are obliged to accept (as modellings of logic for “inconsistent assertors” used in [2] will show). For instance, trouble could of course be obtained through the consistency assumption

\[
P(iv) \quad \vdash_X \sim A / \sim \vdash_X A,
\]

which would restore conclusion 8. But no dialetheician with a modicum of logical sophistication would grant \( P(iv) \) or like principles.

Apart from his main argument, which we have just impeached, Goldstein indicates, largely by appeal to dubious authority, some quite popular supplementary considerations designed to show or lend weight to the idea that contradictions cannot be asserted. As we have dealt with all these considerations elsewhere (especially [3]), we can be brief.

* Contradictions cannot figure in argumentative roles, or at least as hypotheses or assumptions.
Many arguments – Goldstein’s main argument for instance, reductio methods, medieval obligationes – depend for their success on arriving at contradictions, which therefore figure in arguments. Moreover they are, accordingly, quite regularly taken, at least in concealed form, as hypotheses. And in suppositional and “natural deduction” procedures, any contradiction is (correctly) allowed as an hypothesis. Furthermore, contradictions do have consequences; that is often how we find out that they are contradictions. Even were we to give some weight to connexive or holist objections to $A \land \sim A \rightarrow A$, we should be loath to abandon consequences like $A \land \sim A \rightarrow \sim A \land A$, $(A \land \sim A) \land B \rightarrow (A \land B) \land \sim A$, and so forth, for inclusive implication.

* Contradictions (or sentences of contradictory form) do not yield statement because
a. they say nothing, have no content.

But they cannot all say nothing, because different contradictions say different things, and different arguments must be brought to bear against different ones. What is more, a theory of content which reflects these differences, and does assign content to contradictions can be supplied (see [4], appendix).

b. they are meaningless.

But then they would not be able to figure in argumentative roles, in the way we have argued they do, to the extent that they do. What is worse, nor would their negations, all the “anti-contradictory” statements of mathematics. The proposal would entirely cripple analytic reasoning. (For elaboration, and for several other objections, see [5].)

* Contradictions cannot be thought (cf. Wittgenstein).

Even if this were true, it would not follow that they cannot be asserted. Objects such as goods may not be fully comprehensible and intelligible, but assertions about them can be made and used. It is not, however, altogether clear what the claim means. Contradictions can certainly be thought about; lots of logical time has been devoted to such pastimes. And in a sense they can be thought through. They can certainly be considered, contemplated, worried about, and so on. In short, they are proper objects of (many) propositional attitudes. All of which tells against the next claim also:
* One who puts up or persists with a contradiction “does not understand the meaning of his words, and so fails to make an assertion” (p.12).

The intended conclusion does not follow; the assertor may make an unintended statement. More important, the underlying assumption is empirically false. Many people, not just dialetheicians, understand the meaning of the words in the English expression of $R \in R$, and the meanings of the words in the given combination. For they pass all ordinary tests. Should they, having understood the meanings involved, put it up, that does not disqualify them from having understood; they do not retain their understanding only by remaining silent!

They may disqualify themselves in the view of the mainstream logical world if they put it up as more than a hypothesis to be quickly abandoned; but the fault is not strictly one of meaning or understanding, it is one of being swamped in triviality (perhaps, in Aristotelian terms, of babbling). The problem with contradictions, on mainstream perceptions, is not that they cannot be asserted, but that they lead, not nowhere, but everywhere. But no one with a modicum of logical sophistication, not just dialetheicians, would accept this idea, would grant ex falso quodlibet and its scruffy mates.

Notes

1 A theory which can manage to survive on its own, or in isolation (ward), may fail through its intended (epitheoretic), or through its natural, surroundings. So it appears to be, for example, with classically-formulated mathematical theories in natural language settings.

2 Mainstream theory is naturally much worse shaped than paraconsistent when it comes to inconsistent onslaught or entanglement. Those in the know will appreciate that the situation is vastly improved in paraconsistent relevant logics, which offer much superior control over what implies, and what does not imply, what.
References

[1] L. Goldstein, *A problem for the dialetheist*, Bulletin of the Section of Logic, Polish Academy of Sciences 15/1 (1986), pp. 10–14; all quotations are drawn from this article.


