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REMARKS ON REFERENTIAL MATRICES

Sentences describe situations, and from the point of view of pragmat- ics, the situation described by a sentence depends in turn on the context of use, i.e. on the situation in which the sentence is uttered. In general, situations are abstract imaginary entities, but some of them do actually hold; these are called facts. If a sentence $A$ uttered in situation $a$ describes a fact, that is, if the situation described by $A$ (at $a$) actually holds in the real world, then $A$ is said to be true (at $a$). Hence there are at least two ways of defining the meaning of a sentence in pragmatics; first, the meaning of $A$ could be that function from situations into $\{0, 1\}$ which takes the value 1 for exactly those situations in which $A$ is true; second, the meaning of $A$ could be that function from situations into situations which, for an argument $a$, takes the value $b$ exactly in the case when $A$, if uttered in situation $a$, describes situation $b$.

These two notions of meaning are clearly not equivalent. Let us call sentences $A$ and $B$ coreferential if their meanings in the first sense coincide, and synonymous if their meanings in the second sense coincide.

Coreferentially in the case of prepositional calculi is the start point in Wójciicki’s [4], where a new semantic device, called referential matrix, is introduced. (The reader interested in more details may also consult Malinowski’s [2] for a generalization, and Czelakowski’s [1] for an application.) Let us briefly recall Wójciicki’s definition. Where $B = \langle S, F_1, \ldots, F_n \rangle$ is a propositional language (with variables $v_1, v_2, v_3, \ldots$), by a referential algebra (for $B$) is understood any algebra $R = \langle R, f_1, \ldots, f_n \rangle$ similar to $B$, such that for some set $U \neq 0$, $R$ is a subset of the set of all functions from $U$ into $\{0, 1\}$. For any $a \in U$ we put $D_a := \{ r \in R : r(a) = 1 \}$, and let $\mathcal{D} := \{ D_a : a \in U \}$. Then the pair $R = \langle R, \mathcal{D} \rangle$ is a generalized
matrix (cf. [3]), called a referential matrix for \( B \) (over \( U \)). Let \( C \) be a logic, that is, a structural consequence operation in \( B \). \( C \) is said to be self-extensional iff all logically equivalent formulas are intersubstitutable, that is, if \( C(A) = C(B) \) implies \( C(D[A/v_1]) = C(D[B/v_1]) \) for all \( A, B, D \in S \).

It is proved in [4] that it is exactly self-extensional logics which have adequate referential semantics, that is \( C = Cn_R \) for some referential \( R \) iff \( C \) is self-extensional. That not all logics have adequate referential semantics is obviously due to the fact that, in referential matrices, coreferential sentences are considered as if they were synonymous. Malinowski’s generalization in [2] consists in defining so called pseudo-referential matrices to the effect that all logics have an adequate matrix. What I am going to do here is to give a slight generalization of Wójcicki’s notion, which I believe to be much more intuitive than that of Malinowski’s, to exactly the same effect.

Let \( B = < S, F_1, \ldots, F_1 > \) be a propositional language, let \( U \neq \emptyset \) be a set, called the set of situations, and let \( T \) be a subset of \( U \), called the set of facts. By a pragmatic algebra for \( B \) (over \( U \)) we understand one of the form \( P = < P, f_1, \ldots, f_n > \), similar to \( B \), where \( P \) is a nonempty subset of the set of all functions from \( U \) into \( U^{n} \); \( P \subseteq U^{n} \). By a pragmatic matrix for \( B \) over \( P, T \) we mean one of the form \( P = < P, D > \), where \( D := \{ D_a : a \in U \} \), \( D_a := \{ p \in P : p(a) \in T \} \). Let us recall the definition, from [3], of the logic \( Cn_M \) generated in \( B \) by a generalized matrix \( M = < A, D > \), where \( A = < A, f_1, \ldots, f_n > \), \( D \subseteq 2^A \); for every \( X \subseteq S \), for every \( B \in S \),

\[
B \in Cn_M(X) := \forall h \in Hom(B, A) \forall D \in D[hX \subseteq D \rightarrow hB \in D].
\]

**Theorem 1.** For every logic \( C \) in \( B \) there is a pragmatic matrix \( P \) strongly adequate for \( C \), that is, such that \( C = Cn_P \).

Thus, where \( P \) is a pragmatic matrix, \( Cn_P \) need not be self-extensional. Let \( P = < P, D > \) be a pragmatic matrix (over \( U, P, T \), as above) and let \( h \) be a homomorphism from \( B \) into \( P \). We shall say that formulas \( A \) and \( B \) are coreferential in \( P \) with respect to \( h \), in symbols \( A \sim hB \) iff for every \( a \in U, h(A)(a) \in T \) iff \( h(B)(a) \in T \). We shall say that the language \( B \) is extensional with respect to \( P \), in symbols \( P \in Ext(B) \), iff for every \( h \in Hom(B, P) \), for every \( i \leq n, A_i \sim hB_i \) & \( \ldots \) & \( A_K \sim hB_K \) implies \( F_1(A_1, \ldots, A_K) \sim hF_1(B_1, \ldots, B_K) \).
A and B are synonymous in $\mathcal{P}$ with respect to $h$, in symbols $A \approx h_{\mathcal{P}} B$, iff $h(A) = h(B)$; and $B$ is said to be strongly extensional with respect to $\mathcal{P}$, in symbols $\mathcal{P} \in Ext(B)$, if $\sim h_{\mathcal{P}} \subseteq \approx h_{\mathcal{P}}$ for all $h \in Hom(B, \mathcal{P})$. The classes $Ext(B)$ and $\bar{Ext}(B)$ do not coincide; hence, since clearly $\approx h_{\mathcal{P}}$ is always contained in $\sim h_{\mathcal{P}}$, $Ext(B) \subseteq Ext(B)$.

**Theorem 2.** If $\mathcal{P} \in Ext(B)$ then $Cn_{\mathcal{P}}$ is self-extensional.

Where $A = \langle A, f_1, \ldots, f_n \rangle$ is an algebra similar to $B$ and $M = \langle A, D \rangle$ is a generalized matrix, we put $\overline{M} := \langle A, D \cup \{A\} \rangle$. Clearly, for all $M, Cn_M = Cn_{\overline{M}}$.

**Theorem 3.** For every referential matrix $\mathcal{R}$ there is a pragmatic matrix $\mathcal{P} \in Ext(B)$ such that $\mathcal{R} \cong \mathcal{P}$.

**Corollary.** For every logic $C$ in $B$ the following conditions are equivalent:

1. $C = Cn_{\mathcal{R}}$ for some referential matrix $\mathcal{R}$,
2. $C = Cn_{\mathcal{P}}$ for some pragmatic matrix $\mathcal{P} \in Ext(B)$,
3. $C = Cn_{\mathcal{P}}$ for some pragmatic matrix $\mathcal{P} \in Ext(B)$,
4. $C$ is self-extensional.

**References**


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