Andrée Loparic  
Leila Z. Puga

TWO SYSTEMS OF DEONTIC LOGIC

1. Introduction

In this note we outline two systems of deontic logic, one classical and the other para-consistent, denoted by $D^O_2$ and $D^l_2$ respectively. Both are deontic first-order logics with identity.

$D^O_2$ is a classical system of deontic logic, i.e. it is obtained from the classical first-order predicate calculus, by the adjunction to it of the operator $O$ (it is obligatory that) governed by appropriate postulates (rules of inference and axiom schemes). Since $D^O_2$ is based on classical logic, it rules out the so-called moral dilemmas, that is, situations in which one has, for some sentence $A$, that $OA \land O \neg A$ is true, where $\land$ and $\neg$ are the symbols for conjunction and negation respectively (on moral dilemmas, see Barcan Marcus 1980 and Routley and Plumwood 1984). On the other hand, in $D^O_2$ the identity behaves like a deontically obligatory predicate, in the sense that the scheme $x = y \rightarrow O(x = y)$, where $x$ and $y$ are variables, is valid in $D^O_2$. Clearly, this property of identity could be removed, if convenient restrictions were introduced in the axiom for identity.

In $D^O_2$ one has the theorem: $(OA \land O \neg A) \rightarrow B$ and, in particular, $(OA \land O \neg A) \rightarrow OB$. Therefore, a moral dilemma trivializes $D^O_2$. If we want a deontic logic which does not rule out such dilemmas, there are open two distinct possibilities: 1) To weaken the specific postulates for the deontic connectives, and to keep classical logic; this is, for instance, the device suggested by Chellas 1980. 2) To employ in the place of classical logic some kind of non-classical logic, for example a paraconsistent system (see da Costa and Carnielli 1986 and Routley and Plumwood 1984). If

in $D^O_\sim$ we replace classical logic by the paraconsistent system $C^\sim$ (see da Costa 1974), we get $D^\sim_\sim$. This deontic logic does not exclude the moral dilemmas (cf. da Costa and Carnielli 1986).

$D^O_\sim$ and $D^\sim_\sim$ have nice Kripke semantics, and can be modified in several ways. Among other things, they contribute to throw some light on the problem of transparency and opacity of deontic contexts (on opacity and transparency, see Quine 1960).

Our terminology, symbolism, etc. are clear adaptations of those of Kleene 1952 and Shoenfield 1967.

2. The system $D^O_\sim$

The primitive symbols of the language $L^O_\sim$ of $D^O_\sim$ are the following: 1) Logical symbols: $\rightarrow$ (implication), $\neg$ (negation), $\Diamond$ (it is obligatory that), $\forall$ (the universal quantifier), and $=$ (identity). 2) A denumerably infinite set of individual variables. 3) Any arbitrary family whatever of individual constants. 4) For any natural number $n$ greater than zero, a collection of $n$-adic predicate symbols. 5) Parentheses. We define term and formula, introduce the other strict logical symbols and the remaining deontic operators, etc., as usual.

Postulates of $D^O_\sim$, in which capital Latin letters stand for formulas, etc:

- $A_1 : A \rightarrow (B \rightarrow A)$
- $A_2 : (A \rightarrow B) \rightarrow ((A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow C))$
- $A_3 : (\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$
- $A_4 : OA \rightarrow PA$
- $A_5 : O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$
- $A_6 : PA \rightarrow OPA$
- $A_7 : \forall x A \rightarrow A_x[t]$
- $A_8 : x = x$
- $A_9 : x = y \rightarrow (A \rightarrow A_x[y])$
- $A_{10} : \forall x OA \rightarrow O\forall x A$
- $A_{11} : x \neq y \rightarrow O(x \neq y)$
- $R_1 : A, A \rightarrow B/B$ (modus ponens)
- $R_2 : A/OA$
- $R_3 : A \rightarrow B/A \rightarrow \forall x B$
The preceding postulates are subject to the usual restrictions.

Adapting arguments found in Hughes and Cresswell 1968 and in Gallin 1975, we can introduce a Kripke semantics for $D^O_l$, relative to which it is sound and complete. Our semantics gives a clear meaning to our deontic principles, and with its help we can cope with the deontic paradoxes, like those of Ross and of the Good Samaritan (see Hilpinen 1971 and Routley and Plumwood 1984).

3. The system $D^O_l$

The underlying language to $D^O_l$, $L^O_l$, contains the following primitive symbols: 1) Logical symbols: $\rightarrow$ (implication), $\land$ (conjunction), $\lor$ (disjunction), $\sim$ (negation), $O$ (it is obligatory that), $\forall$ (the universal quantifier), $\exists$ (the existential quantifier), and $=$ (identity); equivalence, $\leftrightarrow$, is defined as usual. 2) The non-logical aid auxiliary symbols of $L^O_l$. We easily introduce the standard syntactic concepts in the common way. So, in a certain sense, $L^O_l$ and $L^l_l$ have the same expressive power, with the difference that some defined symbols of $L^O_l$ are primitive in $L^l_l$.

Postulates of $D^O_l$, where $A^\circ$ abbreviates $\sim (A \land \sim A)$ and capital Latin letters stand for formulas:

\[ A_1 : A \rightarrow (B \rightarrow A) \]
\[ A_2 : (A \rightarrow B) \rightarrow ((A \rightarrow (B \rightarrow C)) \rightarrow (A \rightarrow C)) \]
\[ A_3 : A \land B \rightarrow A \]
\[ A_4 : A \land B \rightarrow B \]
\[ A_5 : A \rightarrow (B \rightarrow (A \land B)) \]
\[ A_6 : A \rightarrow A \lor B \]
\[ A_7 : B \rightarrow A \lor B \]
\[ A_8 : (A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow ((A \lor B) \rightarrow C)) \]
\[ A_9 : A \lor \sim A \]
\[ A_{10} : \sim \sim A \rightarrow A \]
\[ A_{11} : B^\circ \rightarrow ((A \rightarrow B) \rightarrow ((A \rightarrow \sim B) \rightarrow \sim A)) \]
\[ A_{12} : A^\circ \land B^\circ \rightarrow ((A \land \land B)^\circ \land (A \rightarrow B)^\circ \land (A \lor B)^\circ) \]
\[ A_{13} : A^\circ \rightarrow (OA)^\circ \]
\[ A_{14} : OA \rightarrow PA \]
\[ A_{15} : O(A \rightarrow B) \rightarrow (OA \rightarrow OB) \]
\[ A_{16} : PA \rightarrow OPA \]
\[ A_{17} : \forall x A \rightarrow A[t] \]
\[ A_{18} : A[t] \rightarrow \exists x A \]
\[ A_{19} : \forall x A^\circ \rightarrow ( (\forall x A)^\circ \land (\exists x A)^\circ ) \]
\[ A_{20} : x = x \]
\[ A_{21} : x = y \rightarrow (A \rightarrow A[x]) \]
\[ A_{22} : \text{If } A \text{ and } B \text{ are congruent formulas (see Kleene 1952, p. 153) or one is obtained from the other by the suppression of vacuous quantifications, then } A \leftrightarrow B \text{ is an axiom.} \]

\[ R_1 : A, A \rightarrow B/B \]
\[ R_2 : A/OA \]
\[ R_3 : A \rightarrow B/A \rightarrow \forall x B \]
\[ R_4 : A \rightarrow /\exists x A \rightarrow B \]

The operators \( F \) (it is forbidden), \( P \) (it is permitted), and \( I \) (it is indifferent) may be defined as functions of \( O \) and \( \sim \), or as functions of \( O \) and the strong negation of \( C = l \) (see da Costa 1974).

It is not difficult to modify the semantics for \( D_O^w \) in order to obtain a Kripke semantics for \( D_O^i \). Relative to this semantics, \( D_O^i \) is sound and complete. Furthermore, schemes like \((OA \land O \sim A) \rightarrow OB\), \((OA \land O \sim A) \rightarrow B\), and \((FA \land F \sim A) \rightarrow B\) are not valid. So \( D_O^i \) does not exclude the moral dilemmas, and ethical systems which do admit such dilemmas as expressing situations that may really occur, can be constructed on it.

4. Concluding remarks

Employing other systems of paraconsistent logic, for example a member of da Costa’s hierarchy \( C_n^w \), \( 0 < n \leq \omega \), we can construct various systems of paraconsistent deontic logic. We also think that paracomplete (see Loparic and da Costa 1984) deontic logics could be relevant in the domain of Ethics. Perhaps, some other changes in our systems would be convenient, for example, the use of conditional obligations instead of unconditional ones.

The prepositional deontic logics corresponding to \( D_O^w \) and \( D_O^i \) are decidable, as it can be proved by the method of filtrations. We observe that the propositional fragment of \( D_O^w \) is equivalent to Chellas’ system \( KD5 \) (see Chellas 1980).
References


State University of Campinas  
Department of Philosophy  
Campinas, SP, Brazil

and

Pontifical University of São Paulo  
Department of Mathematics  
São Paulo, SP, Brazil