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SŁUPECKI'S FRAGMENTARY SYSTEMS

This is an abstract of the paper submitted to Acta Universitatis Wratislaviensis.

In [1] J. Słupecki considers two fragmentary systems S_n and S_n^* of the sentence calculus and gives the set of axioms for them.

Let S be the set of all formulas constructed from propositional variables and the binary functor C. Recall that the set S_n consists of all formulas α ($\alpha \in S$) having the property that the number of occurrences each variable in α is divisible by n. S_n is a system with respect of the rule of detachment and the rule of substitutions.

The matrix

$$\mathcal{M}_n = \langle \{0, 1, \ldots, n-1\},\ \{0\}, C_n\rangle$$

(where C_n is the addition mod n) is a (weakly) adequate matrix for the system S_n (see [1], i.e. $S_n = E(\mathcal{M}_n)$).

J. Słupecki notices that the system S_2 is the bivalued equivalential sentence calculus examined by Leśniewski.

J. Słupecki defines the system S_n^* as follows:

$$S_n^* = S_c \cap S_n$$

where S_c is the bivalued implicational system of the sentence calculus. Let the matrix $\mathcal{M}_n^*$ be the cartesian product of the classical matrix $\mathcal{M}_c = \langle\{0, 1\}, \{0\}, C_c\rangle$[1] and of the matrix $\mathcal{M}_n$. J. Słupecki proved that the matrix $\mathcal{M}_n^*$ is a (weakly) adequated matrix for S_n (see [1]), i.e.

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[1] $C_cab = \begin{cases} 0, & \text{where } a = 1 \quad \text{or } b = 0, \\ 1, & \text{where } a = 0 \quad \text{and } b = 1. \end{cases}$

$$S_n^* = E(\mathcal{M}_n^*) = E(\mathcal{M}_c) \cap E(\mathcal{M}_n).$$

Let $Cn\langle A\rangle$ be the consequence function based on A and on detachment rule and substitutions rule. Further, let $Cn^{-1}\langle A, B\rangle$ be rejected consequence function based on the set A of axioms, on the set B of rejected axioms and on Łukasiewicz's rejected rules.

Definition. A system $\langle S, Cn\langle A\rangle, Cn^{-1}\langle A, B\rangle\rangle$ we shall call a Ł-decidable system iff the following conditions are satisfied (see [2]):

(I) $Cn\langle A\rangle(\emptyset) \cup Cn^{-1}\langle A, B\rangle(\emptyset) = S$,

(II) $Cn\langle A\rangle(\emptyset) \cap Cn^{-1}\langle A, B\rangle(\emptyset) = \emptyset$.

Let A_n, A_n^* be Słupecki's sets of axioms for S_n and S_n^* respectively (see [1]), and let B_n, B_n^* be sets of rejected axioms for S_n and S_n^* $(n \geq 2)$. We accept the following sets of rejected axioms[2]:

$$B_n = \begin{cases} \{[Cp]^0 p,\ [Cp]^1 p, \ldots, [Cp]^{\frac{n}{2}-1} p\}, & \text{wherw } n - \text{even number,} \\ \{[Cp]^0 p,\ [Cp]^1 p, \ldots, [Cp]^{\frac{n-1}{2}-1} p\}, & \text{wherw } n - \text{odd number.} \end{cases}$$

$$B_n^* = \begin{cases} \{C[Cp]^2 pp, [Cp]^2 p\}, & \text{where } n = 2, \\ \{C[Cp]^{n-2} pp, [Cp]^n p, [Cp]^1 p, \ldots, [Cp]^{\frac{n}{2}-1} p\}, & \text{where } n > 2 \text{ and} \\ & n - \text{even number,} \\ \{C[Cp]^{n-2} pp, [Cp]^n p, [Cp]^1 p, \ldots, [Cp]^{\frac{n-1}{2}-1} p\}, & \text{where } n > 2 \text{ and} \\ & n - \text{odd number.} \end{cases}$$

[2]The symbol $[C\alpha]^k \beta$ is defined as follows:

$$[C\alpha]^k \beta = \begin{cases} \beta, & \text{where } k = 0, \\ C\alpha[C\alpha]^{k-1}/\beta, & \text{where } k \geq 1. \end{cases}$$

We proved

THEOREM. *The systems*

$$\langle S, Cn\langle A_n\rangle, Cn^{-1}\langle A_n, B_n\rangle\rangle, \langle S, Cn\langle A_n^*\rangle, Cn^{-1}\langle A_n^*, B_n^*\rangle\rangle$$

are L-decidable.

References

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