Bulletin of the **S**ection of **L**ogic
Volume 13/4 (1984), pp. 230–232

reedition 2008 [original edition, pp. 230–232]

Baláza Biró

ON ISOMORPHIC BUT NOT LOWER-BASE-ISOMORPHIC CYLINDRIC SET ALGEBRAS

Submitted to Journal of Symbolic Logic

Abstract

It is examined in several papers when is it true the isomorphism of two cylindric set algebras is a base-isomorphism.

PROBLEM 1. For which $\mathcal{M} \in CA_\alpha$ it is true that if for $i = 0$ and $i = 1$ $\mathcal{D}_i \in Ca_\alpha, \underline{f}_i : \mathcal{M} \to \mathcal{D}_i$ is an isomorphisms, and $\underline{\text{base}}\ \mathcal{D}_i = V_i$ imply that there exists a bijection $g : V_0 \to V_1$ such that $g = \underline{f}_1 \circ \underline{f}_0^{-1}$ (for notations and notions see [3] and [5]). By the Löwenheim-Skolem theorem this implication is hardly ever true. An important generalization of the base-isomorphisms is the *sub-base-isomorphism* and the *lower-base-isomorphism*, the equivalence relation generated by the relation sub-base-isomorphism.

For lower-base isomorphism the above problem reads as follows.

PROBLEM 2. For which $\mathcal{M} \in CA_\alpha$ it is true that if for $i = 0$ and $i = 1$ $\mathcal{D}_i \in Cs_\alpha\ \underline{f}_i : \mathcal{M} \to \mathcal{D}_i$ is an isomorphism then $f_1 \circ f_0^{-1} : \mathcal{D}_0 \to \mathcal{D}_1$ is a lower-base-isomorphism.

There are sufficient conditions on $\mathcal{M}$ or on the base of $\mathcal{D}_i$ in Theorem 1.3.6 of [4], in [6] and in Proposition 3.4 (3) of [1]. By Corollary 1 of [7] we have the following sufficient condition for Problem 2.

THEOREM 3. *If $\mathcal{M} \in Lf_\alpha$ is countable generated, for every $\underline{n} \in \omega$ $Nr_n\mathcal{M}$ is atomic and $\mathcal{D}_i \in Cs_\alpha^{reg}, f_i \in \underline{Is}(\mathcal{M}, \mathcal{D}_i)$ $(i = 0, 1)$ then $\underline{f}_1 \circ \underline{f}_0^{-1} : \mathcal{D}_0 \to \mathcal{D}_1$ is a lower base-isomorphism.*

Concerning Theorem 3 the following problems arise.

PROBLEM 4. Can some of the conditions of Theorem 3 omitted? Especially

PROBLEM 4 (A). Is the condition '$\mathcal{M}$ is countable generated' omittable i.e. does there exist an Lf_α $\mathcal{M}$ such that for every $n \in \omega$ $Nr_n\mathcal{M}$ is atomic, and for $i = 0$ and $i = 1$ $\mathcal{D}_i \in Cs_\alpha^{reg}, \underline{f}_i \in Is(\mathcal{M}, \mathcal{D}_i)$ such that $\underline{f}_1 \circ \underline{f}_0^{-1}$ is not a lower base-isomorphism?

PROBLEM 4 (B). Is it possible with the above conditions that $\mathcal{D}_0$ and $\mathcal{D}_1$ are not lower base-isomorphic? (Cf. Proposition 3.5 (3), Problem 3 and Problem 4 of [1].)

We prove that the answer to Problem 4 (b) (and hence for Problem 4 (a) is in affirmative as follows.

THEOREM 5. (See [2].) *For every $\alpha \geq \omega$, there exists an χ_1-generated $\mathcal{M} \in Lf$, such that for every $n \in \omega$ $Nr_n\mathcal{M}$ is atomic and for $i = 0$ and $i = 1$ there exists a $\mathcal{D}_i \in Cs_\alpha^{reg}$ and an $f_i \in Is(\mathcal{M}, \mathcal{D}_i)$ such that $\mathcal{D}_0$ and $\mathcal{D}_1$ are not lower-base-isomorphic.*

ACKNOWLEDGEMENTS. We thank S. Shelah and M. Makkai for their much precious help.

References

[1] H. Andreéka, I. Németi, *On cylindric-relativized set algebras*, [5], pp. 131–315.

[2] B. Biró, *On isomorphic but not lower base-isomorphic cylindric set algebras*, submitted to **Journal of Symbolic Logic**.

[3] L. Henkin, J. D. Monk, A. Tarski, **Cylindric Algebras.I**, North-Holland Publishing Company, Amsterdam, 1971.

[4] L. Henkin, J. D. Monk, A. Tarski, *Cylindric set algebras and related structures*, [5], pp. 1–129.

[5] L. Henkin, J. D. Monk, A. Tarski, H. Andréka, I. Németi, *Cylindric Set Algebras*, **Lecture Notes in Mathematics** 883, Springer-Verlag, Berlin, 1981.

[6] J. Larson, *The number of one-generated cylindric set algebras of dimension greater than two*, to appear in **Journal of Symbolic Logic**.

[7] G. Serény, **On regular locally finite-dimensional cylindric set algebras with atomic neat redacts**, manuscript.

Mathematical Institute of the
Hungarian Academy of Sciences