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TOWARDS A FORMAL SEMIOTICS
(abstract)

This is to introduce an outline of a theory of "communication systems" or "systems of signs", called here *codes.* The present theory does not employ any semantic notion, and necessarily does not, for the intended scope of applications (models) is extremely wide. As expressions of a code are thought of not only words, texts, animal or machine signals and the like, but also "signs" without any traditional "meaning" at all: sounds of music, music compositions, dance figures, op-art paintings and their parts; codes in general are merely supposed to have a conventional structure, called grammar, semantics being essential only if *interpreted* codes are dealt with.

The primitives of the theory are the notions of a *meaningful expression*, of the *nonsense*, and of a *substitution*; the set of all the substitutions is said to be the *grammar* of the code. The notion of a category and the relation "x occurs in y" are then definable in terms of these. Substitutions are not mechanical replacements of definite things by other definite things, they are abstract operations on expressions. For example, one may substitute "take off" for "see" in "I saw it" to obtain "I took it off"; observe that "mechanically" the expression "see" does not even occur in "I sat it". Similarly, in tango, we could substitute some figure (of tango!) for another, but substituting a figure of rock-and-roll for it would lead to nonsense-in-some-sense. An outline of the formal construction is presented below. (To make the construction more intuitive, we consider here only substitutions of the form "one expression for one expression". So called simultaneous substitutions are omitted.)

By a *code* we understand any system

$$\mathcal{K} = < \underline{W}, N, \{F_A^B\}_{A,B \in \underline{W}^*} >,$$

where $\underline{W}$ is a nonempty set of so called meaningful *expressions* of $\mathcal{K}$, N is called *nonsense*, $N \notin \underline{W}$, and $\underline{W}^*$ is the set of all *expressions* of $\mathcal{K}$, i.e. $\underline{W}^* = \underline{W} \cup \{N\}$; for any $A, B \in \underline{W}^*$, F_A^B is a *substitution*, i.e. a function from $\underline{W}^*$ into $\underline{W}^*$. The substitutions F_A^B form the set called a *grammar* of $\mathcal{K}$; the grammar is supposed to fulfil the following axioms $(K1) - (K6)$:

$(K1)$ $F_A^B(N) = N$
$(K2)$ $F_N^A(B) = B$
$(K1)$ $F_A^A(B) = B$

Definition 1. Let $A, B \in \underline{W}^*$. The expression A is said to be a *part* of B, or is said to *occur* in B, in symbols $A\varepsilon B$, if there exists a $C \in \underline{W}^*$ such that $F_A^C(B) \neq B$.

$(K4)$ $A\varepsilon B \rightarrow B \not\varepsilon A$
$(K5)$ $F_A^B(C) = C \rightarrow \forall D(D\varepsilon C \rightarrow F_A^B(D) = D)$
$(K6)$ $A\varepsilon B$ & $F_A^C(B) \neq N \rightarrow C\varepsilon F_A^C(B)$.

It may easily be proved that for any expression A, neither $N\varepsilon A$ nor $A\varepsilon N$. These facts, together with the uniqueness of N, point to the rather conventional nature of our nonsense. N represents no concrete thing, it is more like a test of good grammar. The equality $F_A^B(C) = N$ says that the grammar of the code under consideration "prohibits" substituting B for A in C, and nothing more.

Clearly, for any $A, A \not\varepsilon A$; thus Def. 1 defines in fact the relation of being a *proper* part. As other examples of consequences of axioms $(K1) - (K6)$ (which are provably independent), may serve: (I) A becomes nonsense if we substitute N for any part of A, and (II) ε is transitive.

The apparatus just sketched suffices to introduce the general notions of the grammatical category: relative and absolute.

Definition 2. Let $A, B \in \underline{W}$. We put $K_A(B) =_{df} \{C \in \underline{W} : F_A^C(B) \neq N\}$; the set $K_A(B)$ is called the (relative) *category of* A *in* B. A set $K \subseteq \underline{W}$ is a (relative) *category* of $K = K_A(B)$ for some A, B.

Thus the category of an expression depends on the context. It seems to mirror the real situation; as an example compare the grammatical category of the English "green" in "This apple la green" and in "Green apples are sour". But perhaps there are expressions with categories not depending on the context:

DEFINITION 3. A category K is to be *absolute* provided that for any A, B, if $A \in K$ and $A \varepsilon B$, then $K_A(B) = K$.

Observe that (I) $\underline{W}$ is always a category; (II) $A \in K_A(B)$, for all $A, B \in \underline{W}$; (III) for all $A, B \in \underline{W}$ whenever $A \not\varepsilon B$. A code is *regular* if the following condition is satisfied:

(KR) $A \varepsilon B$ & $F_A^C(B) \neq N \rightarrow F_C^A(F_A^C(B)) \neq N$.

Of course, the converse $F_A^C(B) = N \rightarrow F_C^A F_A^C(B) = N$ is true in any code. (KR) says that if we can substitute C for A in B without losing sense, we can also "invert" the operation, i.e. substitute A for C in $F_A^C(B)$; the outcome may be different from the initial B, but it must be meaningful. In other words: if A, B are meaningful expressions of a regular code, than $C \in K_A(B)$ if and only if $A \in K_C(F_A^C(B))$.

Let K_1, K_2 be categories of a code $\mathcal{K}$ and let $K_1 \not\subseteq K_2$.

Then

(i) K_2 is not absolute;

(ii) if moreover $\mathcal{K}$ is regular and $K_2 \neq \underline{W}$ then K_1 is not absolute either.

Making use of the above theorem one can easily prove that any different absolute categories must be disjoint. Following an idea by B. Wolniewicz we call $\mathcal{K}$ *simple* provided that $\mathcal{K}$ is a code with exactly one category. Of course, every simple code is regular.

For any $\mathcal{K}$ the following conditions are equivalent:

(i) $\mathcal{K}$ is simple;

(ii) there are no meaningful expressions A, B, C with $F_A^B(C) = N$;

(iii) $\underline{W}$ is an absolute category.

By a *propositional code* we understand any pair $< \mathcal{K}, \underline{S} >$, where $\underline{S}$ is an absolute category of $\mathcal{K}$ called a category of *sentences*. If there is a designated subset $\underline{T}$ of $\underline{S}$, called the set of *true sentences* then the triple $<\mathcal{K}, \underline{S}, \underline{T}>$ is said to be a *code with semantics* or an *interpreted code*. Now we are going to distinguish a class of codes called *languages*. By a language we would like to understand systems in which reasonings are possible, i.e. ones with some sort of logic; these must contain analytic sentences in $\underline{S}$. Formal definitions are as follows.

Let B be an expression of a code $\mathcal{K}$ and let $A \varepsilon B$. Then the set $\{F_A^C(B) : C \in K_A(B)\}$ is called a *scheme*. Elements of a scheme X are called *realizations* of X. If $<\mathcal{K}, \underline{S}>$ is a propositional code, $B \in \underline{S}$, and $X = \{F_A^C(B) : C \in K_A(B)\} \subseteq \underline{S}$, then X is said to be a *propositional scheme* or a *formula*. If X is a formula of an interpreted code $<\mathcal{K}, \underline{S}, \underline{T}>$ then X is a *tautology* provided that $X \subseteq \underline{T}$, i.e. if all realizations of X are true. A sentence of an interpreted code is *analytic* if it is a realization of a tautology.

A code is a *language* if the set of its analytic sentences is nonempty, i.e. if the code produces at least one tautology. A language is *formal* if the set of its analytic sentences coincides with the set of its true sentences.

For codes with semantics we can reconstruct the notion of entailment, defining it relatively to a class L of meaningful expressions. For any $X \subseteq \underline{W}, A \in \underline{W}$ we put

$$X \vdash_L A =_{df} \ \forall B \in L\ \forall C(F_B^C(X) \subseteq \underline{T} \rightarrow F_B^C(A) \in \underline{T}).$$

If there is a class $\underline{X}$ of absolute categories such that $L = \bigcup \underline{X}$, then $\vdash_L$ is called a *logic*. One can easily verify that $\vdash_L$ is always a consequence relation.

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