1. There seems to be something mysterious about applications of formal systems, including those of logic, to empirical reality. If logic is to be applied to empirical situations, like those described in an ordinary language, then – it seems to some people – its statements cannot be necessary, or analytic, propositions. However, they are both applicable and necessary. This supposed puzzle constitutes a significant part of the problem of philosophical foundations of logic.

To this mind of the present writer, there is no mystery of applications, since any empirical, or even ostensive, predicate can be involved in certain meaning postulates, e.g. “No red is green”, which are both empirical and necessary propositions; empirical as they involve ostensibly defined predications; necessary, as it is enough to know their meaning to state, their validity. The case of logical theorems can be considered at the same footing, logical constants being treated as defined ostensively. Nevertheless, not an individual state of mind but the actual state of scholarly discussions makes something a problem; in the present state of foundational discussions on logic, the “mystery of applications” preserves its vitality, hence any new way of dealing with it proves welcome.

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1 This paper was presented at the conference in Jablonna in the version bearing the title: On the dialogical foundations of logic.

2 A clear exposition of the problem of relations between logic and experience has been given by K. Ajdukiewicz in his essay “Logika a doświadczenie” (1947), contained in the collection “Język i poznanie”, vol. 2, PWN 1965, and also in his “Pragmatic Logic”, Reidel-PWN 1974, pp. 198 ff.
2.1. A new way is offered by dialogic logic as created by P. Lorenzen with collaboration of K. Lorenz. Dialogic logic is an inferential system using only indirect proofs. Proof steps are distributed into two dialogueing parts called “proponent” and “opponent”: the former states the proposition to be discussed and defends it, the latter attacks this proposition, each attack being followed by a move of the proponent which is either a defense or a counterattack.

There are two kinds of rules in dialogic logic: (i) rules of inference, all of them being elimination rules, i.e. prescribing how to drop the main (in the formula in question) logical constant in the process of decomposing the initial proposition; (ii) structural rules, i.e. those which prescribe a sequence of players’ moves, e.g. whether defence is to follow immediately after attack, or may be postponed. The latter kind of rules is crucial for that purpose of dialogical logic which consists in drawing a demarcation line between classical and intuitionistic logic. Elimination rules are related to another purpose of dialogical logic which is to show roots of logic in the ordinary language as used in empirical situations and conversational activities. Within the limits of this paper only the elimination rules, as related to its subject, can be taken into consideration.

2.2. Here are the elimination rules of dialogic logic, each of them designated by the symbol whose elimination the rule deals with, together with the indication whether the rule in question is concerned with defense (D) or refutation (R), i.e. attack.

\( (D.\&\) \) To defined \( A \& B \), prove \( A \) and prove \( B \).

\( (D.\lor) \) To defined \( A \lor B \), either prove \( A \) or prove \( B \) (or both).

\( (R.\neg) \) To refute \( \neg A \), prove \( A \).

\( (R.\rightarrow) \) To refute \( A \rightarrow B \), prove \( B \).

\( (D\rightarrow) \) To defend \( A \rightarrow B \), when \( A \) is proved, prove \( B \).

\( (D.U) \) To defend \( (Ux)A(x) \), prove any instance of \( A(x) \) as required by the opponent, e.g. \( A(b) \).

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\(^3\) See P. Lorenzen and K. Lorenz: “Dialogische Logic”, Wissenschaftliche Buchsellach 1978, Darmstadt. See also relevant essays in: K. Lorenz (ed.), “Konstruktionen versus Positionen: Beiträge zur Diskussion um die Konstruktive Wissenschaftstheorie”; Walter de Gruyter 1979, Berlin – New York. In these books there can be found many other bibliographical references concerning dialogic logic.
To defend \((Ex)A(x)\), prove any instance of \(A(x)\) whatever, e.g. \(A(b)\).

The proponent wins the game, i.e. succeeds in proving that a formula is a theorem of logic, when the opponent in his attempts at refutation necessarily turns inconsistent, either by assuming two contradictory statements (each in a separate move), or by assuming just this sentence which is required by the proponent to defined his position\(^4\). The opponent wins the game if he saves himself from the inconsistency, while all the possibilities of defense have been exhausted by the proponent.

3.1. Before we discuss how the dialogical elimination rules are related to our everyday experience, let us state the following epistemological principle: the construction or the acquisition of new concept by means of ostensive definitions or analogous devices requires certain presupposed notions (the term “presupposed” is used here as less committing then the term “a priori”). For instance, when we learn a new concept, i.e. the meaning of a predicate, by means of an ostensive definition, then what must be presupposed is the notion of a set which is necessary to grasp the function

\(^4\)Here are examples of dialogical game-proofs of logical theorems, with “O” standing for the opponent and “P” for the proponent.

<table>
<thead>
<tr>
<th>0</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(p \rightarrow \neg(p))</td>
</tr>
<tr>
<td>0</td>
<td>(\neg(p) \rightarrow p)</td>
</tr>
</tbody>
</table>

As to the last move, 0 has two options: if he chooses \(p\), then he gets inconsistent by assuming both \(p\) and (as above) \(\neg p\); if he chooses \(q\), then he provides \(P\) with the assertion required for the defense of the conditional in question.
of the predicate as predicate: otherwise the example object presented in the ostensive definition would not be regarded as a representative of predicate’s extension (still more is presupposed in ostensive definitions, but let this example suffice).

Now, it is assumed in the philosophy of dialogic logic (as understood by the present author) that the meaning of a logical constant can be defined in certain situation contexts that operate like ostensive definitions. If so, we are to describe how this is carried out, and to find presupposed notions. The notions we find are: that of obligation and that of proof, or demonstration, together resulting in a more involved notion: the obligation to prove.

3.2. Let us note that the whole of our social life is built upon a structural framework (a skeleton, so to say) of mutual obligations, or duties, and their converses called rights. Without such an obligation framework there would be amorphous collections of individuals instead of solid social structures. These observations are relevant for the problem of foundations of logic since they show how fundamental the idea of obligation is for human thinking. Rewards and punishments as applied in the teaching of a particular obligation are intelligible for the person being taught only when the general notion of obligation is provided.

The idea of obligation contributes to definitions of logical constance in the following way. First, let us note that it is possible to combine an ostensive definition with a contextual one; such a combination appears when we define an operation (which requires a contextual definition) in a situation in which the truth of operation arguments is being perceptually ostended. Such a procedure may be called: the definition by a situational context. Second, let us note that the fulfilling of the duty of proof by a person \( x \) can be rewarded by a sign of approval (not necessarily a verbal sign) of a person \( y \), and this sign is to mean that the sentence in question has got proven. That the arguments are proven is (in the situations like those here discussed) just what the persons involved can see with their eyes; this fact is crucial for problem “logic and experience”, since it yields an empirical interpretation of logical operations as due to their empirical origin in situational contexts.

3.3. Let us discuss some examples. The logical constant ASSERTION, e.g. in the form “... is true”, can be defined in the following situational context. A person \( x \) has the duty to person \( y \) to prove \( A \). If \( x \) manages
to ostend the state of affairs denoted by $A$, as observable for both parts, then $y$’s approval combined with an utterance like “$A$ is true”, or “$A$ has got proven”, completes the situational definition of assertion (as a concept being introduced to the vocabulary of $x$). The operation DENIAL can be situationally defined by the context in which someone denies $\neg A$ by the assertion of $A$ (compare rule ($R, \neg$)).

Let a person $x$ ought to prove DISJUNCTION “either $A$ or $B$”, where $A$ and $B$ are perceptual statements; $x$ proves, say, $A$ by ostending the state in question, and thereby he wins the approval of a competent person $y$ (e.g., a native speaker, or a teacher of logic); on another occasion, $x$ proves the same disjunction by producing $B$ and wins $y$’s approval as well. This experience becomes a paradigm of using the logical constant “either ... or”, what for a amounts to entering into possession of a rule like ($D, \lor$).

Such a process of learning logical operation (as expressed by logical constants) may be completed by the situations in which the lack of success of proving is followed by the disapproval of a competent person. Thus we can obtain a pragmatic counterpart of truth tables in which the truth of an argument would be represented by the perception of an observable state of affairs, the lack of truth – by the lack of required perception, while for the compound proposition the truth and its lack would be represented by the approval and the disapproval of a competent person, respectively.

Another example: in some everyday situations we learn how to refute a conditional $A \rightarrow B$ by producing the assertion of $A$ together with the denial of $B$; e.g., when I do not agree that a dog bites if it is teased, I tease it and show that it fails to bite. Hence, whoever tries to refute a conditional has to start from proving its antecedent, and this is just what the rule ($R, \rightarrow$) tells us. Therefore, the antecedent having been proved, the rule of defense, viz. ($D, \rightarrow$) demands that the consequent be proved.

4. Thus, when defining logical constants by situational contexts, we fix their meaning with inference rules, the original situational contexts being paradigms of application or rules. Theorems of logic are analytic propositions whose validity is totally dependent on the meaning of their logical constants; in this sense they are independent of experience. However, the logical constants themselves have such a meaning that its empirical interpretation derives from the very mode of defining them, viz. defining by situational contexts. Now, if somebody sees a puzzle of application, it is up to him to state propositions about analyticity and empirical applicability.
of logic which would seem to be inconsistent with each other.

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