The relevance of metamathematical researches for philosophy of mathematics is an indubitable matter. In the paper I shall speak about implications of metamathematics for general philosophy, especially for classical epistemological problems. Let us start with a historical observation concerning Hilbert’s programme, the first research programme in metamathematics as a separate study of formal systems. This programme was strongly influenced by epistemological considerations. In fact, Hilbert wanted to secure all classical mathematics (including “suspected” parts of the set theory) against inconsistencies and this aim had been achieved with the help of finitary consistency proof. Hilbert’s claim is like the Cartesian project of reduction of all concepts to clareae et distinctae ideae. The epistemological commitment of original Hilbert’s programme may be treated as a preliminary heuristic motivation for looking for epistemological implications of metamathematical results. It seems reasonable to examine in this respect three so-called limitative theorems: the first Gödel’s incompleteness theorem (if $S$ is consistent, then $S$ is incomplete), the second Gödel’s incompleteness theorem consistency of $S$, cannot be proved in $S$, providing that $S$ is consistent) and Tarski’s undefinability theorem (the set true propositions of $S$ is undefinable in $S$); “$S$” stands for formal system containing elementary number theory. From the above-mentioned theorems we obtain an important, from philosophical point of view, conclusion: semantics of $S$ cannot be expressed in $S$.

Now, I shall present three applications of limitative theorems to the analysis of classical epistemological problems.

(1) *The analytic/synthetic distinction.* Let us suppose: (a) analytic = a priori, (b) mathematic = logic, (c) analytic propositions $=d^f$ propositions
provable (or disprovable) in logic (Frege’s notion of analysity). Copi [4], [5] argues that by the first Gödel’s theorem there are arithmetical truths non-provable in logic, so they have to be regarded as non-analytic. Copi does not maintain that non-provable arithmetical truth are synthetic a priori claiming only that (a) – (c) should be carefully reconsidered. I note also some other standpoints concerning analysity, motivated by Gödel’s theorem. DeLong [6] argues that formula expressing consistency of $S$ is synthetic (as unprovable) and a priori (as being necessary true, if true at all). Borkowski [3] offers the distinction between analytical in syntactic sense (Frege’s concept of analysity) and analysicals in semantic sense (true in all models); analysicals in the first sense are a proper subset of analysicals in the second sense. And, finally, Turquette [7] argues that metamathematical results have no importance for the distinction in question. Thus, we have a relatively simple problem and almost all possible solutions motivated by two limitative theorems.

(2) The coherence theory of truth. The coherence theory of truth is, perhaps, the most important rival of classical theory of truth. The canonical statement of coherence theory is the following: a proposition, say $A$, is true, if and only if, $A$ is coherent with an earlier accepted body of knowledge. It seems, that, according to the well-known intuitions of coherentionists, we are perfectly entitled to the replacement of “coherent” by “consistent”. Also we can suppose that “the body of knowledge” ($K$) contains elementary number theory and can be formalized. Nothing specially wrong for coherentionism results from the first incompleteness theorem; it seems, even, that an adherent of coherence theory is able to use this theorem for the strengthening of his position. He can say that if $K$ is consistent, then both $K \cup \{A\}$ and $K \cup \{\neg A\}$ are consistent too. Next, he may observe that under one interpretation of vocabulary of $K$ and $A$, $A$ is true and under another interpretation $\neg A$ is true; conventionalism which is, normally, a part of coherence theory gives here a considerable help. On the other hand, quite disastrous consequences for coherence theory follow from the second incompleteness theorem. Consistency is not only the “essence” of truth, but also its criterion. If consistency of $K$ cannot be proved in $K$, then consistency-criterion is not quite reliable, because it is impossible to prove it universally; obviously, the defence of coherence position by an appeal to consistency of $K$ proved in an oversystem of $K$ is unsuccessful, as the latter is more “suspected” with respect to consistency. The most
profound argument against coherence theory seems to follow form Tarski’s theorem: the set of true propositions of $K$ in the classical sense (providing that truth in Tarski’s sense = truth in classical sense) is greater than the set of true propositions of $K$ in the sense of coherence theory (providing that truth as defined by syntactic consistency of $X =$ truth in sense of coherence theory).

(3) *Esse* = *percipi*. Ajdukiewicz [2] pointed out an interesting analogy between semantics and epistemology. If one intends to speak (in epistemology) about thoughts and objects, he has to use a suitable language, similar to semantical metalanguage, and if an epistemologist restricts himself to speak about thoughts only, he needs a language, like syntactical metalanguage. According to Ajdukiewicz, Berkeley in his famous dictum *esse* = *percipi* intended to say something about relation between thoughts and world, but he expressed *esse* = *percipi* in the improper, syntactic-like language. Suszko (unpublished paper delivered at a meeting of the Polish Philosophical Society in Cracow, 1962) inferred from Ajdukiewicz’s analogy very strong conclusion. Suppose that *esse* is a semantic term and *percipi* is a syntactic one. Under this assumption, Berkeley’s dictum is simply false, because semantics of the language cannot be expressed in its syntax.

All examples end with strong philosophical conclusions such as the possibility of synthetic a priori propositions, the defence of classical theory of truth or the refutation of subjective idealism. But one must be very careful while regarding the above mentioned analysis as simple inferences from limitative theorems. For these theorems are not about analytic or synthetic propositions, classical or coherence theories of truth, *esse* or *percipi*; limitative theorems are about provability, consistency, completeness, truth in Tarski’s sense and the like. The matter does not consist in a replacement of words by words, although such replacements form an essential part of analytical procedures—these procedures are rather paraphrases (cf. Ajdukiewicz [1]). Ajdukiewicz pointed out that there are two ways of justification of paraphrases. The first way of justification appeals to meanings postulates and the second to the phenomenological analysis of meaning in Husserl’s sense; and of course there are various compromise positions. I do no claim any univocal solution of this serious metaphilosophical dilemma (I think that such a solution is impossible to get) and I limit myself to the observation that complications in paraphrases gradually grow from (1) to (3). Relatively easily, I think, we can agree that “provable in logic” is
adequate representation for “analytic in Frege’s sense”, but decision that “truth in classical sense” may be replaced by “truth in Tarski’s sense” meets the well-known objections. And, of course, considerably much more serious problems are involved, if we replace “language of esse” by “semantical metalanguage”.

Examples (1)–(3) represent three great classical problems of epistemology: (1) the problem of source of knowledge in the methodological sense, (2) – the problem of truth, and (3) – the problem of limits of knowledge. These questions may be analyzed independently, but we can look for – by use of metamathematics – some global epistemological perspective. I think that such perspective may be found and be very illuminating. Let us agree that the analysis of validity of $K$ is a business of epistemology and, in particular, that it attempts to answer to the famous Kantian question: *quod iure*? We can expect on the basis of analogy with metamathematics that an answer should be given in a language stronger than the language of $K$; and, of course, this latter language cannot be reduced to the language of $K$. Next, the language of epistemological analysis exceeds the language of $K$ as regards expression; so the hermeneutics of epistemological language is irreducible of the hermeneutics to $K$. This analogy explains some important points relevant for metaphilosophy, i.e. limitations of scientism. I think that certain paraphrases of metamathematical results make epistemological hermeneutics results make epistemological hermeneutics more accessible in comparison with the traditional ways of philosophical thinking. In any case, discussion of epistemological matters ignoring metamathematical results seem to be hopeless in the same sense as discussions about determinism without any appeal to the quantum theory.

References


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