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BASIC INTUITIONS OF NON-FREGEAN LOGIC

Roman Suszko introduced a broad class of languages into the literature of logic. In honour of L. Wittgenstein Suszko named these languages W-languages. Syntax, semantics and consequence operations in these languages are based on the famous ontological principle: whatever exists is either a situation, or an object, or a function. The distinguishing property of W-languages is that they contain: sentential and nominal variables, an identity connective and an identity predicate. The intended interpretation of W-languages is such that: sentential variables range over universum of situations, nominal variables range over universum of objects. All other symbols in these languages, except sentential and nominal variables, are interpreted as symbols of some functions defined in both universum of situations and universum of objects. The identity connective corresponds to the identity relation between situations, and the identity predicate corresponds to the identity relation between objects.

In W-languages non-Fregean logic is generated by the rule Modus Ponens and by logical axioms divided into three groups: axioms for truth-functional connectives (classical), axioms for quantifiers, and axioms for identity connective and predicate. For interpretations of W-languages we assume the following semantic principles:

1. Sentences and names form disjoint syntactic and semantic categories. The principle was not assumed by G. Frege. For Frege, sentences are names for special kind of objects which he called logical objects.
2. The two-valuedness of logic, i.e. any logical sentence is either true or false. There are functions v defined on sentences whose values are either true or false.
3. Following Frege and Wittgenstein we assume that sentences and names

have references. If φ is any logical sentence or name, then there is exactly one referent of φ, which is denoted by $h(\varphi)$. The refernt of φ is what is given by φ.

4. The extensionality of logic in Frege's sense, i.e. if two expressions have the same referents, then they are interchangeable in any context without changing the referent of this context.
5. If two sentences α, β are interchangeable in any sentential context without changing the logical value of this context, then these sentences have the same referents.

Frege believes that in view of the postulates from 2. – 5. we have to identify the referents of sentence with their truth values i.e. $v(\alpha) = h(\beta)$, where v is a truth-valuation or logical valuation, and h is an ontological valuation. The principle that:

> "all true and similarly all false sentences describe the same state of affairs, that is, they have a common referent"

Suszko called the semantic version of the Fregean axiom. In his formalization of Wittgenstein's Tractatus Logico-Philosophicus, Suszko proves that it is possible to adopt semantic postulates 1. – 5. without adopting the Fregean axiom. Following Wittgenstein, Suszko calls the semantic correlates of sentences; situations. The starting point of Suszko's program is the assumption that the semantic correlates of sentences are different from logical values, i.e. $v(\alpha) \neq h(\alpha)$. In my paper, I will only discuss the most important part of W-languages containing only sentential variables, connective and possibly quantifiers binding sentential variables.

By a sentential calculus we shall understand a couple $(\underline{L}, Cn)$, where $\underline{L}$ is a sentential language and Cn is a structural consequence operation on $\underline{L}$. Interpretations of the sentential language $\underline{L}$ satisfying conditions 3. and 4. are arbitrary homomorphisms of the language into an algebra similar to it. Obviously, not every algebra $\underline{A}$ similar to the sentential language $\underline{L}$ is a formal representation of universum of situations, and naturally not every homomorphism $h \in Hom(\underline{L}, \underline{A})$ is a semantic interpretation of $\underline{L}$.

If we want to treat $(\underline{L}, Cn)$ as a logic of situations, then there must be at least some truth-functional connectives in the language $\underline{L}$. One of the main assumptions of the ontology of Tractatus Logico-Philosophicus is that there are truth-functional connectives in a language of logic. Logic of situations should not contain any quantitative and structural assumptions on universum of situations, except that true sentences describe different situations than false sentences do, so universum of situations has at least two elements. In defining the sentential calculus $(\underline{L}, Cn)$ we determine the meaning of logical constants; but not only, as it may also happen that by assuming some semantic principles some quantitative and structural conditions on universum of situations may become applicable. For example, if the language includes truth-functional connectives as the only logical connectives, then according to the assumed semantic principles, any structure which is the interpretation of any theory in this language is two-element Boolean algebra. To avoid any restrictions on universum of situations, Suszko introduced an identity connective as a non-truth-functional connective to the language of the classical sentential calculus.

In [1], the completeness theorem for non-Fregean logic is proved. Philosophical intuitions concerning non-Fregean logics can be found in [3], [4], [5]. Non-Fregean logic is extensional, logically two-valued, but is not two-valued ontologically, because it does not follow from the theorems of logic alone how many semantic correlates of sentences there can be. Incidentally, some non-Fregean theories, i.e. theories based on non-Fregean consequence operation, contain a theory of modality. To compare non-Fregean theories with modal theories we suppose the rule of translations:

$$\alpha \equiv \beta // \Box(\alpha \leftrightarrow \beta)$$

In the literature on W-languages, the following theories are considered: WBQ, WTQ, WHQ. They are axiomatic strengthenings of non-Fregean logic. Notation:

W- for Wittgenstein
B- for Boolean algebra
Q- for quantifiers
T- for topological
H- for Henle's algebra

On the basis of the thesis from Tractatus Logico-Philosophicus of L. Wittgenstein:

5.141 If p follows from q and q form p, then they are one and the same proposition,

it is natural, in formalizing the ontology of the Tractatus, to consider the WTQ-theory described as follows:

$$WTQ = Cn(\{Gn(\alpha \equiv \beta) : (\alpha \leftrightarrow \beta \in Cn(\emptyset)\})$$

where $Gn(A)$ = the set of all generalization formulas with set A. The theory WTQ includes the following theorems:

(1) $\Box 1 \equiv 1$
(2) $\Box p \leq p$
(3) $\Box\Box p \equiv \Box p$
(4) $\Box(p \wedge q) \equiv (\Box p \wedge \Box q)$
(5) $\forall p \Box \alpha \equiv \Box \forall p \alpha$
(6) $\exists p \Diamond \alpha \equiv \Diamond \exists p \alpha$.

These theorems allow the interpretation of the square connective as an interior operation in a Boolean topological algebra, whose bounds given by the quantifiers satisfy conditions (5), (6). The tractatus thesis:

5.5303 Roughly speaking: to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing.

So we assume the principle as follows:

(7) $\forall p \forall q((p \equiv q) \equiv 0 \vee (p \equiv q) \equiv 1)$

The theory WHQ is simply the theory WTQ with axiom (7) added. The sentential constants $1, 0$ are the unit and zero of the Boolean algebras of situations. We call 1 – the necessary situation, and 0 – the impossible situation. In Boolean theories, let us introduce in W-language the following connective $\leq$ defined as

$$(p \leq q) \equiv ((p \rightarrow q) \equiv 1).$$

We read formula $p \leq q$ as: situation p is included in situation q, or situation p involves situation q, or situation q occurs in situation p. In all theories WBQ, WTQ, WHQ we can define the following philosophically important connectives:

(8) $PWp \equiv ((\neg(p \equiv 0) \wedge \forall q((p \leq q) \vee (p \leq \neg q))$
(9) $RWp \equiv (p \wedge PWp)$
(10) $MFp \equiv \forall q(q \rightarrow p \leq q) \wedge \forall r(\forall q(q \rightarrow r \leq q) \rightarrow (r \leq p)))$

By the introduced notation, these connectives have the following meaning:

PW – possible world
RW – real world
MF – meet of all facts.

According to (8), (9), (10)

r is a possible world equals to r is a possible situation and for every situation q or $\neg q$ occurs in p;
r is a real world equals to r is a possible world and r is a fact;
r is the meet of all facts equals to r contains all facts and nothing else but facts.

Using these connectives we can describe several properties of algebras of situations and sets of facts. In particular, the connective MF can be used to interpret Wittgenstein's thesis:

1. The world is everything that is the case.
1.1 The world is the totality of facts, not of things.

Theorems of theories WBQ, WTQ, WHQ are formulas:

(11) $\exists pMFp$
(12) $MFp \wedge MFq \rightarrow (p \equiv q)$

These formulas say that there exists only one situation, which is the meet of all facts. They do not imply whether the meet of all facts is, or is not a fact itself, thus whether it is a real situation or not. The theories WTQ may be identified under this rule translation $\alpha \equiv \beta // \Box(\alpha \leftrightarrow \beta)$ with well-known modal theories $S_4\Pi$, $S_5\Pi$, respectively.
These theories have been considered by Kit Fine in [2].

References

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