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HOW DISJUNCTIVE SYLLOGISM CAN BE SEEN
AS A FALLACY OF RELEVANCE

Anderson and Belnap’s thesis that Disjunctive Syllogism ($DS$), $(A \lor B)$ & $\sim A \rightarrow B$, commits a fallacy of relevance, has often been challenged. Copeland, to focus on one recent example, not only claims (in [1]) that ‘no account of premiss/conclusion relevance has been produced which shows $(DS)$ to commit a fallacy of relevance’ (p. 333), but ‘doubts whether any account of premiss/conclusion relevance could have been’ (p. 333). It is immediate from the (first) claim and another extravagant assertion (p. 327, partly italicized statement) that ‘Anderson and Belnap have not shown that the exclusion of $(DS)$ from their entailment system has anything to do with the matter of relevance’. The latter claim is surely false: Anderson and Belnap have at least shown (in [2]) that the addition of $DS$ to their system $E$ results in variable-sharing irrelevance. For the addition of $DS$ to $E$ reinstates Lewis’s paradox argument and so yields such variable irrelevance as $p$ & $\sim p \rightarrow q$ (cf. $E$ pp. 164–5).

More important, Copeland’s main claims are also mistaken. In particular, an account of premiss/conclusion relevance is easily produced from the literature on entailment (in fact known to Copeland) which reveals $DS$ as, derivatively, a fallacy of relevance. As Anderson and Belnap explain
informal discussions of implication or entailment have frequently demanded “relevance” of \( A \) to \( B \) as a necessary condition for the truth of \( A \to B \), where relevance is now construed as involving some “meaning content” common to both \( A \) and \( B \). This call for common meaning content comes from a variety of quarters. Nelson ... says that implication “is a necessary connection between meanings”; Duncan-Jones ... that \( A \) implies \( B \) only when \( B \) “arises out of the meaning of” \( A \); Baylis ... that if \( A \) implies \( B \) then “the intensional meaning of \( B \) is identical with a part of the intensional meaning of \( A \)” ... (Ep.32).

Inclusion of content affords an account (one among several) of “Common meaning context” which brings out all these calls for relevance in implication – where the content of \( A \), \( c(A) \), is explicated as the class of worlds (or set-ups) where \( A \) does not hold: \( c(A) = \{ a \in K : I(A, a) \neq 1 \} \). Then \( A \to B \) iff \( c(B) \subseteq c(A) \), that is, \( B \) is part of the meaning of \( A \). Details of this account are sketched, for the first degree, in [3], and are extended to the higher degree in [4]. (Since the account turns on semantics for relevant logic, friends of irrelevance will, of course, try to find objections to it. Also the account is, as it stands, system-dependent, and derivative upon independent choice of system. But there are other matters, discussed elsewhere, especially [5]. The account is an account).

Now simply define content relevance – premiss/conclusion relevance is really a misdescription – as follows: \( A \) is content relevant to \( B \) iff \( B \) is part of the content of \( A \). Then every implicational thesis of a relevant logic such as \( E \) satisfies content relevance, as can be demonstrated using the semantics for the logic, e.g \( \vdash E A \to B \) iff \( A \) is content relevant to \( B \). But \( DS \) violates content relevance, and so constitutes, in Anderson and Belnap’s terms, a fallacy of content relevance. For 
\[
c((A \lor B) \& \sim A) = c(A \lor B) \cup c(\sim A) = (c(A) \cap c(B)) \cup c(\sim A) = (c(A) \cup c(\sim A)) \cap (c(B) \cup c(\sim A)).
\]
Hence \( c(B) \subseteq c((A \lor B) \& \sim A) \) only if \( c(B) \subseteq c(A) \cup c(\sim A) \). But \( c(B) \nsubseteq c(A) \cup c(\sim A) \), since for some inconsistent worlds \( A \) and \( \sim A \) both hold but \( B \) does not. Therefore \( c(B) \nsubseteq c((A \lor B) \& \sim A) \), exposing \( DS \)’s fallaciousness. (Note that the relation defined is transitive but, naturally, not symmetric. Symmetric, but non-transitive, content relevance relations
are readily defined: e.g. $C_1(A, B) =_{DF} (c(B) \subseteq c(A)) \& (c(A) \subseteq c(B))$ and, corresponding to Copeland’s semantic overlap, $C_2(A, B) =_{DF} c(A) \cap c(B) \neq A$. Then if $\vdash E A \rightarrow B$ then $C_1(A, B)$ and $C_2(A, B)$, but not necessarily conversely. Naturally $\sim C_1((A \lor B) \& \sim A, B)$. Contrary then to Copeland (Cp. 330, p. 332), $E$ answers to several accounts of relevance. Also note that what amount to content measures on canonical models have been used throughout).

Content relevance also puts an end to other parts of Copeland’s trouble finding. Consider, to illustrate, his

doubt whether any account of premiss/conclusion relevance could be produced which showed the claim ... that $A \lor B \rightarrow C$ is valid if and only if $A \rightarrow C$ and $B \rightarrow C$ are both valid ... to be correct (p. 333, rearranged).

Content relevance removes such doubts. For $A \lor B \rightarrow C$ is valid iff generally $c(C) \subseteq c(A \lor B)$, i.e. iff $c(C) \subseteq c(A) \cap c(B)$, i.e. iff $c(C) \subseteq c(A)$ and $c(C) \subseteq c(B)$, i.e. iff $A \rightarrow C$ and $B \rightarrow C$ are valid. (In fact, each of a variety of semantics for relevant logics delivers the same conclusion).

Even though content relevance does the requisite work, Anderson and Belnap do not escape scot-free. Firstly, they offer no such account of content relevance, and even if they had it would require underpinning by independent arguments for the choice of $E$, since similar accounts work for a variety of logics including irrelevant ones. Secondly, they give no explanation of how it is that $DS$ is a fallacy of relevance. In fact, they seem to think (E p. 165) that they have shown that it ‘commits nothing less than a fallacy of relevance’ through ‘an “independent proof” of the invalidity of’ $DS$. In an oblique fashion (which has eluded several readers) this is so, as regards the first of the two accounts of fallacies of relevance they offer, the use account. For if $DS$ is invalid then it fails on whatever corresponding subscripted natural deduction system is used to keep track of use of hypothesis. On the second, variable-sharing, account of relevance, $DS$ does not violate relevance at all, since $(A \lor B) \& \sim A$ and $B$ share variables through the common occurrence of $B$.

The use account is (also, without further ado) only of limited value, since accounts of the same type can reflect a wide range of logics including even irrelevant logics, in particular, as Copeland points out (p. 330), classical logic, that end point of irrelevance. This point — along with other
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troubles in Anderson and Belnap’s use of relevance – was explained in [6] and is much elaborated in [5], chapter 7, where the limited and derivative role of relevance in characterizing entailment is also argued. Observe however that the situation with Distribution, \( A \land (B \lor C) \rightarrow (A \land B) \lor (A \land C) \), is not quite as bad as Copeland Tries to make out (p. 329). A special subscripting rule for Distribution is not essential since the principle can be removed in a fairly natural way, by amending disjunction rules. While it is true that subscripting rules to guarantee such principles as \( DS \) could be added, such principles would violate separation desiderata, by combining in one rule several different connectives).

The variable sharing account is similarly limited (unless extended): it provides only a weak necessary condition on relevance, which quite a variety of systems can satisfy, some of them (such as connective logics) including \( DS \) as a theorem. To surmount the problem, Copeland would like to see variable sharing of antecedent and consequent also as a sufficient condition for relevance, thus exonerating \( DS \) from any criticisms as a fallacy of relevance. But such a proposal is unacceptable, since it would at the same time exonerate such paradoxes as \( A \rightarrow B \rightarrow A \) and \( A \rightarrow \sim A \rightarrow C \) as fallacies of relevance, though they yield in one detachment step fallacies of relevance.

The variable-sharing account admits, however, of extension to a third account, the involvement account Copeland ‘pieces together’ (p. 333), under which \( DS \) is a fallacy of relevance. The account, not produced by Anderson and Belnap, is however illustrated by their actual procedure with \( DS \) (cf. ‘the “independent proof” of the invalidity of’ \( DS \), E p. 165, which is made good E p. 174, and which is what shows that \( DS \) ‘commits a fallacy of relevance’). The principle \( DS \) is tantamount – after distribution – to a wff, \( (A \land \sim A) \lor (B \land \sim A) \rightarrow B \), which is correct only if a component \( A \land \sim A \rightarrow B \), which does commit a (variable-sharing) fallacy of relevance, is correct. In this sense, \( DS \) involves a fallacy of relevance; namely it is tantamount to a statement which is correct only if an irrelevant statement is correct. The technical notion of involvement involves ties with content relevance; for if \( A \rightarrow B \) involves a fallacy of relevance then \( A \rightarrow B \) is content irrelevant. For suppose \( A \rightarrow B \) involves a fallacy of relevance. Then there is some \( C \) tantamount to \( A \rightarrow B \) such that if \( C \) is correct so is \( D \rightarrow E \), but \( D \) and \( E \) fail to share a variable. In this event however, invalidity transfers from \( D \rightarrow E \) to \( C \) and thence to \( A \rightarrow B \); so \( C(B) \nsubseteq c(A) \), and \( A \rightarrow B \) is content irrelevant. There are
some evident difficulties with the involvement account as an explanation of a genuine fallacy; namely that, at least without independent explanation, it assumes notions of equivalence and correctness furnished by the system (or systems in the vicinity of the system) whose adequacy is what is being defined. (Were material equivalence, used, for instance, every implication would involve a fallacy of relevance).

Copeland arrives at the right conclusion that ‘Anderson and Belnap’s rejection of (DS) relies on a new, third, notion of relevance’ (p. 332) but by a defective route, highlighted by straight textual misinterpretation (cf Ep. 165) which leads Copeland to conclude:

Thus Anderson and Belnap are not rejecting the disjunctive syllogism on the ground that its conclusion is irrelevant to its premiss, but rather that two constituents of the premiss (the disjuncts of $A \lor B$) are not relevant to one another. Thus their rejection of the disjunctive syllogism has little to do with their rejection of the disjunctive syllogism has little to do with their demand for an analysis of entailment which takes into account the (putative) fact that relevance between conclusion and premisses is essential to sensible argumentation (p. 332).

None of this is true. The $DS$ inference ‘commits nothing less than a fallacy of relevance’ (Ep. 165), which implies that the conclusion is irrelevant to the premiss (see Ep. 17ff). Copeland manages to attribute to Anderson and Belnap the very opposite of their position by misconstruing the claim, in their restriction of the rejection of $DS$ ‘to the case in which the “or” is taken truth functionally’, that ‘perhaps always when the principle is used in reasoning one has in mind an intensional meaning of “or”, where there is relevance between the disjuncts’ (Ep. 165); from which ‘it emerges’, according to Copeland, ‘that the fallacy of relevance committed is that the disjunctive syllogism licences the inference of $B$ from $\sim A \land (A \lor B)$ when there is no “relevance between the disjuncts” $A, B$’ (p. 332). It does not. What emerges is that where the disjunction in $DS$ is intensional, there is relevance between the disjuncts (since $\sim A \rightarrow B$ is assumed); which says nothing about where the fallacy of relevance in “truth-functional” $DS$ is located.
References

[1] B. J. Copeland, *The trouble Anderson and Belnap have with relevance*, *Philosophical Studies* 37 (1980), pp. 325–33. All page references without further citation are to this article.


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