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ON DEFINING MOISIL’S FUNCTORS IN \( n \)-VALUED ŁUKASIEWICZ PROPOSITIONAL LOGIC

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Gr. C. Moisil defined in his paper [1] the notion of \( n \)-valued Łukasiewicz algebra introducing a family of unary functions \( \sigma_1^n, \ldots, \sigma_{n-1}^n \), which can be interpreted in sentential calculus as a special kind of modal functors:

\[
v(\sigma_k^n \alpha) = \begin{cases} 1 & \text{if } v(\alpha) \geq \frac{k}{n-1} \\ 0 & \text{otherwise.} \end{cases}
\]

In what follows we define the functions \( \sigma_1^n, \ldots, \sigma_{n-1}^n \) in the \( n \)-valued Łukasiewicz propositional calculus (with implication and negation as the only connectives). It may be shown that

\[
\sigma_k^n = N\sigma_{n-k}^n N,
\]

which makes it possible to restrict ourselves to the construction of \( \sigma_i^n \) for \( i \leq \frac{n}{2} \) only.

The construction will be carried out on the grounds of a sequence of formulas defined inductively this way:

\[
A_3(\alpha) = CN\alpha \alpha
\]

\[
A_{k+1}(\alpha) = CN\alpha A_k(\alpha).
\]

A very important property of \( A_n \)’s can be formulated by means of the equality
\[ v(A_m(\alpha)) = \min(v(\alpha) \cdot (m - 1), 1). \]

By virtue of this equality it is possible to prove that the functors \( \sigma^n_1 \) can be defined as follows:

\[
\begin{align*}
\sigma^n_1 \alpha &= \text{df } A_n(\alpha), \\
\sigma^n_k \alpha &= \text{df } \sigma^n_{s} A_{m+1}(\alpha) \quad (\text{for } 1 < k \leq \frac{n}{2}),
\end{align*}
\]

where

\[
m = \max(j : j \cdot (k - 1) \leq n - 1)
\]

and

\[
s = \begin{cases} 
  n - 1 & \text{if } km \geq n - 1, \\
  km & \text{otherwise}.
\end{cases}
\]

References